A COMPLEX SYSTEMS SCIENCE PERSPECTIVE ON WIRELESS NETWORKS *

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Abstract The paper targets a future world where all wireless networks are self-organising entities and in which the predominant mode of spectrum access is dynamic. The paper explores whether *the behaviour of a collection of autonomous self-organising wireless systems* can be treated as a *complex system* and whether *complex systems science* can shed light on the design and deployment of these networks. We focus on networks that self-organise from a frequency perspective to understand the behaviour of a collection of wireless self-organising nodes. Each autonomous network is modelled as a cell in a lattice and follows a simple set of self-organisation rules. Two scenarios are considered, one in which each cell is based on cellular automata and which provides an abstracted view of interference and a second in which each cell uses a self-organising technique which more accurately accounts for interference. We use excess entropy to measure complexity and in combination with entropy we gain an understanding of the structure emerging in the lattice for the self-organising networks. We show that the self-organising systems presented here do exhibit complex behaviour. Finally, we look at the robustness of these complex systems and show that they are robust against changes in the environment.

Key words Excess entropy, robustness, self organising wireless networks, dynamic spectrum access.

1 Introduction

This paper focuses on the application of systems science and complexity science to mobile and wireless communication systems. In particular, it seeks to explore whether future mobile and wireless communication can be treated as complex systems and seeks to uncover insights that can be gleaned from this treatment.

Traditional mobile networks are planned in a highly centralised manner. A good example of this centralised planning can be seen in the use of spectrum. Spectrum is the life-blood of any wireless system. Mobile operators acquire spectrum from a central authority (e.g. a national regulator) and subsequently carefully plan their networks to make the best use of this spectrum. Cellular networks, for example, exploit the fact frequencies can be spatially reused, provided certain conditions are met, to make sure that maximum *coverage and capacity* can be extracted from the available spectrum. This is one example of many. There are very many different operating parameters of a mobile network which are optimised and fine-tuned to maximise network performance.

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A number of advances in technology have disrupted the ability to be able to coordinate and fine-tune everything centrally. On the spectrum side, the traditional approach to obtaining spectrum from a central authority has been challenged through the field of dynamic spectrum access [1],[2]. Techniques have been developed to opportunistically access spectrum, to lease swathes of spectrum for short periods or to trade spectrum between different entities. This results in a much more dynamic environment in which centralised planning becomes challenging. In addition, advances on the infrastructure side are also pushing away from centralised approaches. Dense large-scale uncoordinated deployment by the end users of millions of transceivers (each targeting the coverage of a certain local area), especially in urban environments, mean that the mobile operator does not oversee a coordinated roll out of network infrastructure with a subsequent careful setting of all operation parameters. All of this drives the requirement for self-organising functionality within the transceivers as central planning becomes increasingly infeasible. As a result of these changes, many algorithms exist which support network self-organisation.

The purpose of this paper is to look towards a future world in which all wireless networks are self-organising entities and in which the predominant mode of radio frequency spectrum access is dynamic, and to ask the question whether it is possible to capture the behaviour of such a world in a systematic manner. More specifically the paper explores whether such a self-organising and highly dynamic world can be treated as a complex system and whether complex systems science can shed light on the emergent properties of these kinds of networks or offer new insights that can be used in their design and deployment.

One of the most widely accepted definitions of complex system, is that of "a system in which large networks of components with no central control and simple rules of operation give rise to complex collective behaviour, sophisticated information processing, and adaptation via learning or evolution" [3]. This view resonates with our description of future mobile networks. Of course more formal metrics are needed to determine that a system does in fact exhibit complex behaviour and this issue will be addressed in the main body of the paper.

Work does exist which treats mobile communication networks as complex systems [4][5][6][7]. However, the above mentioned studies focus on human dynamics and social interactions via mobile phones, and they all adopt a network science perspective. In our work, we instead focus on wireless networks from the physical infrastructure perspective; further the angles from which we tackle the problem are of an information theoretical and bio-inspired/self-organising nature.

The paper does the following. It begins by taking a simple approach of modelling a self-organising network (or collection of self-organising networks) using cellular automata, and explores the resulting behaviour to determine whether it can be considered to be a complex entity. To do this we adopt a measure of complexity which is related to organisational aspects, and in particular to the difficulty of describing organisational structure [8]. The cellular automata model, while insightful, does not capture reality. Hence the paper goes on to build a more realistic model of self-organising systems which, for example, take real transmission and propagation issues into account.

The paper makes the following key contributions:

1. It recognizes the possibility that complex systems science can provide a systematic means of understanding the future self-organising, dynamic wireless networks;

2. It recommends a complexity metric that successfully captures the amount of structure of the resulting behaviour of a collection of self-organising networks;

3. It makes the argument that the complex behaviour exhibited by a collection of self-organising wireless networks has attractive properties in terms of robustness to changes

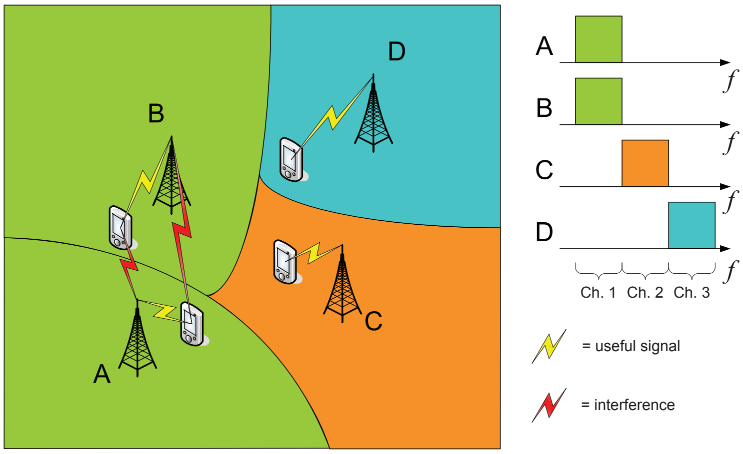


Figure 1: Neighbouring systems simultaneously using the same frequency can interfere with each other.

in the environment.

2 A Simple Approach to Modelling Self-Organising Wireless Networks of the Future

Wireless networks can self-organise on the basis of a wide range of parameters. We begin by very simply focusing *on self-organisation from a frequency perspective alone.* We justify this on the basis that autonomous frequency planning will be one of the predominant and defining features of the future networks that are described in the introduction - i.e. because in these networks spectrum is accessed in a dynamic fashion.

Spectrum is the life-blood of wireless communication networks. It is divided into a finite amount of orthogonal channels, which are used by radio transmitters to send information over the air. Two or more wireless connections that are simultaneously active in close geographical proximity can interfere with each other if the transmissions use the same frequency as depicted in Figure 1, thus leading to the need of a channel allocation mechanism. This is a well studied topic, [9], [10], [11], [12], [13] and hence we note upfront that our aim is **not** to make a contribution to the plethora of autonomous self-organising frequency assignment techniques that already exist, but rather *to study the overall behaviour of the collection of autonomous self-organising systems* and to determine whether these systems exhibit complex behaviour.

Our approach is to model each independent entity as a cell in a lattice which is following a simple set of self-organisation rules and to study what happens to the lattice as the system as a whole strives, from a frequency perspective, to self-plan.

We look at both ideal and realistic lattice formations, the former as mathematical nicety and a means of laying the foundations and the latter as a more realistic example of how networks are deployed. We observe the self-organising behaviour within the lattice to determine whether complex behavioural patterns exist. To do this it is necessary to define some metrics for determining whether the resulting behaviour is complex. We argue that a combination of entropy and complexity metrics is appropriate for our study.

We consider two different sets of rules to model the self-organising frequency assignment behaviour (i.e. to model the behaviour in each cell of the lattice). In the first instance we model each individual system using the concept of cellular automata. We use simple rules based on proximity of neighbouring cells within the lattice to indicate that interference has occurred. Cellular automata are mathematical models for systems consisting of large numbers of simple identical components having local interactions. As such, they provide a natural framework to model communication systems composed by a multitude of simple devices that have to fulfil quality of service/experience requirements in a distributed manner. The cellular automata model, as will be shown, can achieve a non-trivial interference free frequency allocation. More importantly the cellular automata model allows us to make the first steps in identifying complex behaviour patterns.

As stated already the cellular automata approach is based on a highly abstracted definition of interference. To make the situation more realistic, we subsequently adopt a self-organising model for each of the cells that is cognisant of signal transmission and propagation issues, and hence embodies more realistic measures of interference. Here too we show that self-organising solutions emerge that exhibit complex behaviours.

Before proceeding, it is worth noting that a number of approaches studied in complex and adaptive systems science have been successfully applied to the self-organising dynamic frequency assignment problem. In particular, reinforcement learning based techniques have been one of the favorite choices for dynamic channel selection applications [14, 15]. For a review on the use of multi-agent reinforcement learning techniques for adaptive wireless networks the reader is referred to [16].

The remainder of the paper is laid out as follows. Section introduces the complexity and entropy metrics on which our analysis is based. Section presents the cellular automata model and provides an analysis of the behaviour of self-organising systems within regular and irregular lattices. Section explores the more realistic scenario based on real interference calculations and on irregular lattice structures only. Again the behaviour of the set of self-organising systems is explored the conditions under which complex behaviour emerges are identified. Section provides an overall analysis of the implications on channel allocation stability of the identified complex behaviour and Section concludes.

3 Complexity and entropy

It is known from complex systems science literature that complexity and entropy are two distinct quantities. Entropy does not capture the correlation and structure of a system, rather its disorder and inhomogeneity [17]. For studies on the relation between complexity and entropy, readers can refer to [18] and [19]. The general behaviour of the complexity vs. entropy relation is expected to be first increasing and then decreasing (see e.g., [20]); in particular:

- highly regular systems, which present identical structure at all levels, are expected to have low complexity and low entropy;
- *complex systems*, which present non-repeating structure at multiple levels, are expected to show high complexity and intermediate entropy;
- random systems, which present no structure at any level, are expected to show low complexity and high entropy.

We use *excess entropy* to measure complexity for both regular and irregular cells deployment. Excess entropy can be expressed in different ways [17]. The form we utilize in this paper is the convergence excess entropy E_C, which is obtained by considering how the entropy density

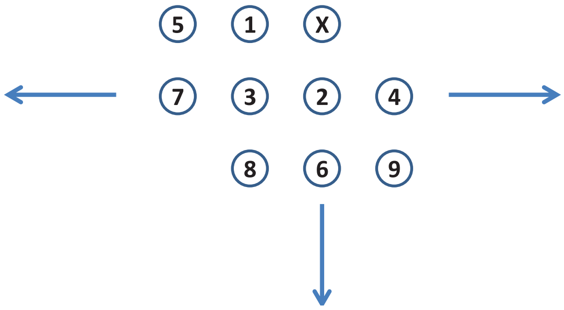


Figure 2: Target cell (X) and neighbourhood templates for 2D conditional entropies (ideal lattice)

estimates converge to their asymptotic value h. This form of excess entropy belongs conceptually to the category of complexity measures related to organisational aspects, and in particular it relates to the difficulty of describing organisational structure [8]. We decided to choose it as measure of complexity, as it respects the general expected complexity vs. entropy behaviour, and it applies naturally to a discrete set of frequencies available at different cells.

3.1 Ideal lattice

In two dimensions the entropy density h can be expressed as [17]:

$$h = \lim_{M \to \infty} h(M), \tag{1}$$

where $h(M)$ is the entropy of the target cell X conditioned on the cells labelled 1, 2, ..., M in Figure 2. The cell numbers in figure indicate the order in which the cells are added to the template. Cells are added to the template in order of increasing Euclidean distance from the target cell X. In case of a tie, the leftmost cell is added first. The limit above exists if the system is translation invariant, and our regular lattice scenario is translation invariant because the cells are homogeneous.

Then, the excess entropy E_C is defined as:

$$E_C = \sum_{M=1}^{\infty} (h(M) - h). \tag{2}$$

The ordering scheme used in this paper is one out of the many possible options. Clearly the entropy estimate does not depend on the particular order in which cells are added to the template because of the limit in (1). It should also be noted that the convergence to h is monotonic, i.e. $h(M') \leq h(M'')\ \forall M' > M''$. However, the ordering scheme can affect E_C. As we use the complexity measure to capture the spatial structure of a channel allocation, which is determined by the interference received by neighbouring cells, the Euclidean distance based ordering is the most reasonable choice. It is worth noting that the template we adopted slightly differs from the one used in [17] as in our case the interactions can extend across more than two lattice sites, i.e. across more than two rows in the lattice.

3.2 Realistic lattice

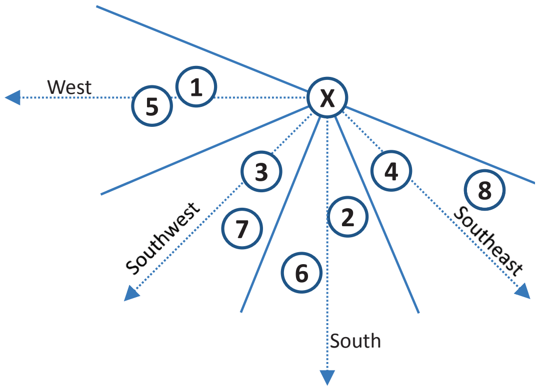


Figure 3: Target cell (X) and neighbourhood templates for 2D conditional entropies (realistic lattice).

We generalized the neighbourhood template for 2D conditional entropies in the case of a realistic cell deployment. In this case we take into account both the Euclidean distance and cardinal directions. Cells are added to the template according to a hierarchical sorting scheme. First the cardinal direction is considered in the following order: West, South, South-west, South-east. Then, for each sector cells are added in order of increasing Euclidean distance. Hence, $h(1)$ denotes the entropy of the target cell X conditioned on the closest cell in the West sector; $h(2)$ denotes the entropy of the target cell X conditioned on the closest cell in the West sector and the closest cell in the South sector, etc. (see Figure 3). It is worth noting that $h(M') \leq h(M'') \; \forall M' > M''$.

3.3 Complexity and entropy in self-organising wireless networks

The remainder of the paper analyses the relationship between complexity and entropy within the context of frequency allocation. In particular, we study the complexity-entropy behaviour for different frequency allocation configurations resulting from:

- Centralized frequency planner;
- Self-organised frequency allocation;
- Random frequency allocation.

We first study the case of an ideal cells deployment. To study the self-organising frequency allocation we model the problem of channel assignment within the framework of cellular automata. In Section we present an algorithm that, under certain conditions, evolves the cellular automaton to an interference-free allocation of the channels in a finite number of steps. We then apply the complexity and entropy measures discussed in Section to study the resulting system-wide behaviour and we show that both centralized and random frequency allocations are characterized by zero complexity, while the self-organised frequency allocation exhibits complexity values higher than zero.

In Section we extend our study to a more realistic scenario based on real interference calculations and on irregular lattice structures only. We present a simple algorithm that uses local information and adaptations to minimize interference among neighbouring cells. The resulting self-organising frequency allocation is analysed using the complexity and entropy measures discussed in Section . Once again we show that the random frequency allocation is characterized by zero complexity, while the self-organised frequency allocation exhibits complexity values higher than zero. Here we did not consider a centralized frequency allocation as it is not suited for a realistic irregular deployment.

4 Frequency Assignment and Cellular Automata

A cellular automaton is a discrete model that consists of a finite dimensional lattice of cells, each in one of a finite number of states. Each cell interacts with a subset of cells, called neighbourhood. At any time instant, each cell updates its state based on the previous states of the cells in its neighbourhood.

A model based on the combined use of cellular automata and learning automata has been applied to channel assignment problem [21][22]. This model allows agents to autonomously discover the mapping between situations and actions through a mechanism of trial and error. We will show that our simple cellular automata-based algorithm can achieve interference-free frequency channel allocation, just relying on very limited local information and without assuming learning capabilities. The simplicity of our model allows us to focus on the main objective of this paper: the study of the overall behaviour of the collection of autonomous self-organising wireless networks to determine whether these systems exhibit complex behaviour.

To model the frequency assignment problem, we consider a two-dimensional cellular automaton: each cell represents a self-organising wireless system, the state of a cell is the frequency selected by the system and its neighbourhood is the set of systems that interfere with the cell when using the same frequency. At this level of abstraction a cell can represent a single wireless node or a whole wireless networks. The key goal in autonomous frequency assignment is that each cells gets a frequency assignment that does not cause interference to others. In the simple cellular automata approach interference only exists if direct neighbours in the lattice have the same frequency. This is a simplistic approach but nonetheless captures the basic concept of frequency reuse distance.* In our study we rely on the two most used neighbourhood functions, namely the Von Neumann and the Moore neighbourhoods. In the remainder of the paper the terms frequency and channel are used interchangeably.

Let us denote by N the number of available channels and by s_i the state of i-th cell. Available channels can be determined from a database or through sensing. Time is slotted and all cells can measure the interference caused by the transmission of adjacent nodes on all channels c_k, $\forall k \in \{1, 2, \ldots, N\}$. We define a cell as active at time t if the cell detected a variation on the measured interference on any channel c_k in the previous time slot. Only active cells at time t can update their state; also, after updating its state, an active cell will remain inactive for the two subsequent time slots. At the beginning of each time slot, each active cell i stops transmitting ($s_i = 0$) and initializes a random timer with period less than the duration of the time slot. When the timer expires, each active cell updates its state, i.e. it selects a channel and starts transmission. The cell chooses the channel randomly among the channels which have not been selected by the subset S of its neighbours that changed state in the previous and current time slot. In practice this means that the cell selects a channel whose interference has not increased in the previous and current time slot.

If each cell behaves according to rules defined above, the evolution of the cellular automaton can be formalized in Algorithm 1.

*The frequency reuse distance is the minimum distance between two transmitters such that transmissions on the same frequency do not interfere with each other.

Algorithm 1 Channel Assignment.

```
Select one random cell n
A_0 = {n}
s_n = 0
Wait until random times expires
s_n ← channel with the lowest interference
t = 1
while A_t ≠ ∅ do
    UpdateCells ( A_t , A_{t−1})
    t = t + 1
    A_t = neig(A_{t−1}) −A_{t−1} − A_{t−2}
```

```
function neig(X)
    return the list of neighbours of all the cells in X
```

```
function UpdateCells(B, C)
    Z = ∅
    for all b ∈ B do
        s_b = 0
    Initialize a timer for each cell in B with period less than duration of the time slot
    while B ≠ ∅ do
        D ← cells in B with the minimum clock period
        for all d ∈ D do
            s_d ← channel with the lowest interference among the channels not used by neig(d) ∩ Z and
            neig(d) ∩ C
        Z = Z ∪ D
        B = B − D
```

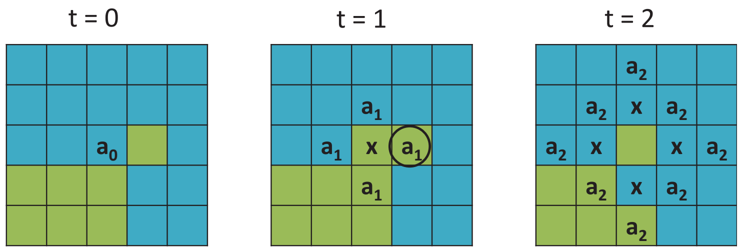


Figure 4: Von Neumann neighbourhood case. Green and blue are the only two colours, i.e. channels, available. We denote with X cells that changed their state in the previous time slot; a_t denotes cells which are active at time t. The decision of the cell circled in figure only depends on the interference resulting from the activity of cell X. Hence, this cell will select the blue channel even if the majority of its neighbours are using the same frequency.

We now focus on the conditions that guarantee the convergence of the cellular automaton to an interference-free allocation of the channels. Let us first examine the Von Neumann neighbourhood function. In this case, only $N = 2$ channels are necessary to achieve an interference-free assignment of the channels. The algorithm starts at time $t = 0$ with a random cell n that stops transmitting and, after a random period, selects the channel c_1 with the lowest interference. Cell n cannot be activated until $t \geq 3$. The $4-$neighbours of cell n will be the only active cells at time $t = 1$. As they all detected an increase of the interference in channel c_1, they will all select the remaining channel c_2. These cells cannot be activated until $t \geq 4$. At time $t = 2$, only 3 of the neighbours of each previously active cell are active. Once again, as they all detected an increase of the interference in channel c_2, they will all select channel c_1. It is important to notice that, in the case of the Von Neumann neighbourhood, active cells do not interfere with each other. Hence, they can update their state synchronously or asynchronously. In other words, there is no need to use a timer. Figure 4 depicts the evolution of the cellular automaton for the Von Neumann neighbourhood. It should be noted that each cell can be activated only once.

In the case of the Moore neighbourhood, it can be verified that the set of active cells evolves according to concentric squares, as depicted in Figure 5. In this case, active cells can interfere with each other. Hence, active cells cannot simultaneously update their state if we want to guarantee convergence to a collision-free allocation of the channels. Formally, the probability that $|D| = 1$ has to be equal to 1. Moreover, the number of available channels N necessary to achieve a collision-free channel assignment has to be greater than or equal to 6. In fact, at each time slot t each active cell has to select a channel which has not been previously selected by the subset S of its neighbours that changed state in the previous and current time slot. As the maximum cardinality of S is 5, $N = 6$ channels are necessary to ensure that such channel exists (see Figure 5). Moreover, for an R-by-R lattice the number of time slots required to converge to an interference-free channel assignment is $T = \max([x + 1, y + 1, R - x, R - y])$, where x and y are the coordinates (row and column) of the starting cell, with $0 \leq x \leq R - 1$ and $0 \leq y \leq R - 1$. For any value of R, the minimum of T occurs when $x = y = \lfloor (R/2) \rfloor$.

We now analyse the probability P of convergence to a collision-free channel assignment for the Moore neighbourhood, when the probability that $|D| = 1$ is less than 1, i.e. when two or more active adjacent cells update their state simultaneously. To this end, we model the random clock generator as an $M-$sided die, where each of the M possible periods occurs with

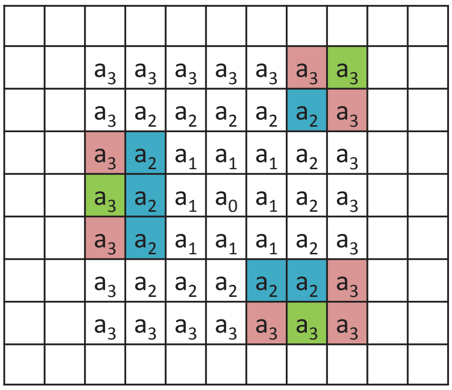


Figure 5: Moore neighbourhood case. We denote by a_t active cells at time t. At time $t = 3$, only cells a_3 are active. Blue coloured cells and light-red coloured cells represent the subset S for each green cell. In particular, for each green cell, blue coloured cells are the subset of its neighbours which changed their state in previous time slot; light-red cells are the subset of its neighbours which changed their state in the current time slot. For any t, $|S| \in \{3, 5\}$.

equal probability. Obviously, the probability that two or more active cells update their state simultaneously decreases when M increases. Also, for a given M, the probability of convergence P is a decreasing function of the dimension of the lattice R. To study this relationship, we run 10^3 independent simulations for a range of values of R. In each case, we select the starting cell that minimizes the number of iterations T for the given lattice dimension: for a given R, the number of iterations T is $\lceil (R/2) \rceil$. Figure 6 shows $\ln(P)$ versus T for different values of M.

When $M = 10^3$ the convergence probability P quickly decreases when T (and R) increases. However, the average number of cells that collide with one or more of their neighbours after a stable channel allocation is reached is negligible (see Figure 7). For example, when $R = 10^2$, only 9 out of 10^4 cells experience interference in the case of $M = 10^3$. As for previous experiments, the average is computed by running 10^3 independent simulations for each value of M and R. When $M = 10^5$ the convergence probability P is greater than 0.99 for any value of T (and R). Accordingly, the average number of cells colliding with one or more of their neighbours is approximately 0. When $M = 10^4$ the convergence probability P is greater than 0.89 for any $R \leq 50$. Moreover, even for larger values of R, the average number of cells experiencing interference is always less than 1.

We shall now present an argument which explains the Gaussian behaviour of the probability P as a function of T.[†] The number of active cells after $k \geq 1$ iterations is equal to $8k$. Before the $k+1$'th iteration, we observe that the shell of active cells which are about to update their state can be divided in small groups of three neighbours where 'things can go wrong'.[‡] Denote by p the probability that such a group makes a correct choice of channels. These groups of three are almost independent, in the sense that the probability of a correct choice of channels in a group of six consecutive active cells is about p^2. Since the total number of active cells before

[†] A Gaussian function is a function whose logarithm is a quadratic function.

[‡] At each iteration $k > 1$ there are also 8 groups of four neighbours which can simultaneously update their state. However, as the correlations induced by this few larger groups are very weak, we will only consider the groups of three cells in our subsequent analysis.

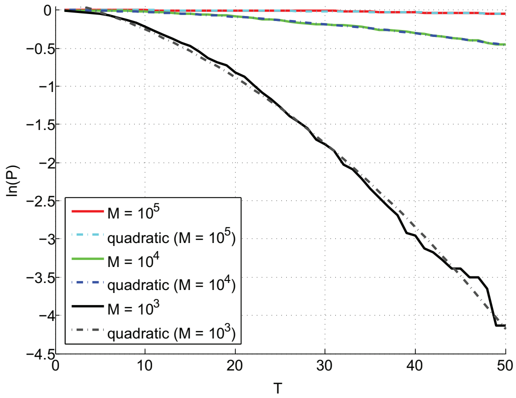


Figure 6: Probability of convergence to a collision-free channel assignment as a function of T and M. The plot also shows the quadratic approximation for P.

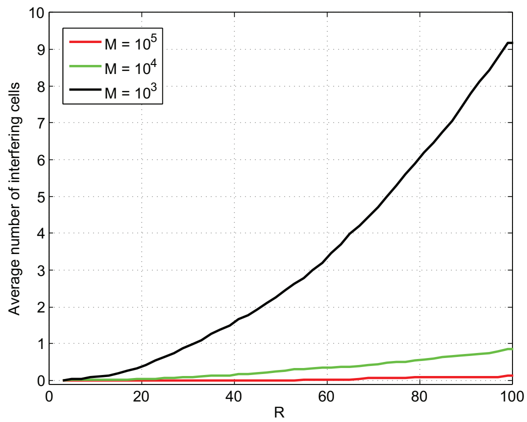
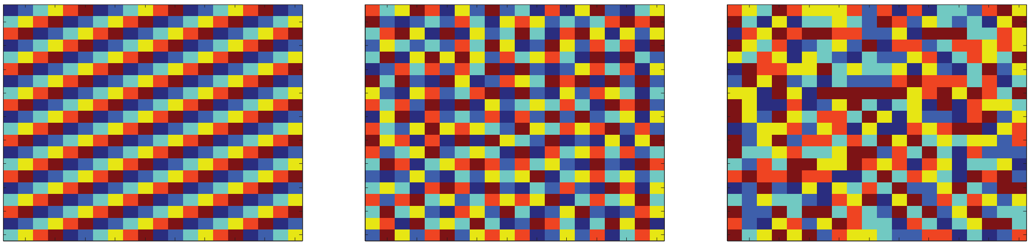


Figure 7: Average number of cells that collide with one or more of their neighbours as a function of R and M.



(a) (b) (c)

Figure 8: (a) Regular channel assignment, $E_C = 0$ and $h = 0$. (b) Channel assignment resulting from the self-organising algorithm described in Section , $E_C = 2.04$ and $h = 1.29$. (c) Random channel assignment, $E_C = 0$ and $h = 2.58$.

iteration $k+1$ is $8(k+1)$, we have that the number of groups of three neighbours 'where things can go wrong' is $8(k+1)/3$. Thus the probability of a correct choice of channels before iteration $k+1$ is approximately $p^{8(k+1)/3}$. In other words, there exists a positive number $q = p^{8/3}$ such that the probability of a correct choice of channels before iteration $k+1$ goes like q^k. Because the iterations are independent from each other, the probability of a correct choice of channels after T iterations is given by $q \cdot q^2 ... \cdot q^T$. Thus:

$$P \sim q^{1+\cdots+T} = q^{T(T+1)/2},$$

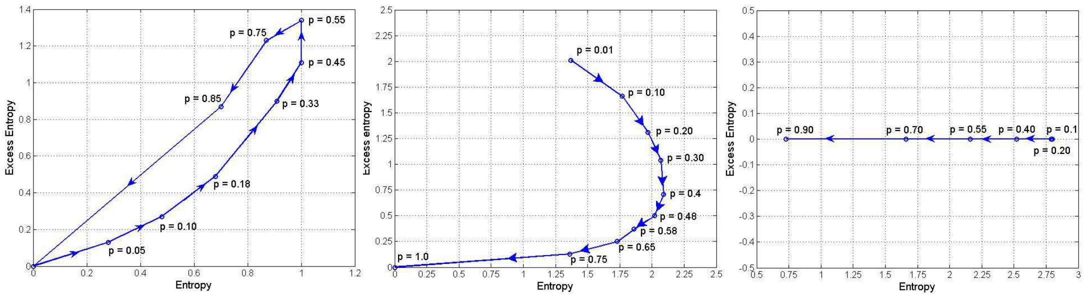
or equivalently, $\ln(P) \sim -\frac{T(T+1)}{2}|\ln(q)|$.

4.1 Complexity and entropy measures for regular cells deployment

It is worth recalling at this stage that the aim of the paper is not to propose a self-organising algorithm for frequency allocation. Instead it is to look at the behaviour of self-organising wireless systems. In our case this system is one which organises on the basis of frequency.

We can now study the global network-wide behaviour with respect to the complexity of the channel allocation matrix. Figure 8(a) shows an example of channel allocation matrix resulting from the regularly spaced assignment of $N = 6$ channels. This is a typical example of the frequency allocation resulting from a centralized frequency planner in the case of homogeneous cells. Figure 8(b) shows an example of the channel allocation matrix resulting from the algorithm described in Section . Finally, a random allocation of $N = 6$ channels is shown in Figure 8(c).

We estimated E_C and h according to definitions given in Section for the three types of channel assignments in Figure 8 by using $10^4 \times 10^4$ matrices. For the type of channel assignment depicted in Figure 8(a), the entropy estimates are $h(M) = 0$, $\forall M = \{1, 2, \ldots, 6\}$. Hence, $E_C = 0$ and $h = 0$. This is consistent with the crystal-like completely ordered structure of the channel allocation matrix. For the random channel assignment matrix shown in Figure 8(c), the entropy estimates are $h(M) = 2.58$, $\forall M = \{1, 2, \ldots, 6\}$. Hence, $E_C = 0$ and $h = 2.58$. As the channel assignment matrix is completely disordered, the entropy is the maximum possible for $N = 6$ channels. Finally, in the case of a channel assignment matrix of the kind represented in Figure 8(b), the entropy estimates are $h(M) = 1.29$, $\forall M = \{4, 5, 6\}$. Hence, $h = 1.29$ and the resulting E_C is 2.04. The channel assignment emerging from self-organisation exhibits a certain amount of structure, which neither the centralized nor the random channel allocation can reach. It is this resulting structure that is of interest to us, in that it can potentially play a role in how networks are designed.



(a) (b) (c)

Figure 9: Complexity-entropy trajectory parameterized over the percentage p of inactive cells for: (a) regular channel assignment; (b) channel assignment resulting from the self-organising algorithm described in Section ; (c) random channel assignment.

4.2 Complexity and entropy measures for irregular cells deployment

We can also look at structure that emerges in an irregular deployment. Figure 9 shows the impact of irregular cells deployment on the complexity-entropy trajectory. In particular, in this figure we highlight the effect of deactivating an increasing percentage p of cells. In the case of the centrally-planned frequency allocation matrix (see Figure 9(a)), the random deactivation of cells disrupts the completely ordered structure of the channel allocation matrix. This introduces an increased level of disorder in the system. Also, as the channel allocation matrix is no longer the repetition of an identical basic pattern, we observe a corresponding increased level of complexity (in fact, we could think of complexity as something measuring the degree of nontrivial, or in other words, *intelligent structure* present in a system). As the number of inactive cells increases the overall randomness and complexity start decreasing to zero (in correspondence to a completely inactive network). For the random channel assignment matrix shown in Figure 9(c), overall the entropy decreases when the percentage of inactive cells increases. However, the amount of structure, as measured by the excess entropy, is constantly zero. Finally, the complexity of the channel assignment matrix resulting from self-organisation decreases when the percentage p of inactive cells increases (see Figure 9(b)). In other words, the more regular the cells deployment, the more structured the channel assignment emerging from self-organisation.

Before we can establish whether we can make use of these insights, we need to develop a more realistic model of interference in wireless networks.

5 A More Realistic Scenario

The cellular automata model of the independent self-organising entities works at a basic level. Interference is modelled based on simple lattice neighbour rules. In this section of the paper we advance the work and model more realistically self-organising systems using rules which better capture real measures of interference, by taking real transmission and propagation issues into account. The model is based on recommendations from a prominent communication standard (3GPP). The goal of this section is mainly to provide some concrete evidence that our framework is valid in real world situations too, and not only interesting from a theoretical viewpoint. Each self-organising entity, i.e. each cell in the lattice, runs an algorithm which aims to reduce intercell interference (of the cells in the lattice) by limiting the reuse of the same frequency by neighbouring cells.

To compute the impact of the transmission of neighbouring cells, a common metric adopted in wireless telecommunications is the Signal to Interference plus Noise Ratio (SINR), which measures the quality of the received signal relative to interference from other signals and noise. It should be noted that the SINR is measured at the receiver. An accurate estimate of the SINR can be calculated by including an appropriate propagation model. In fact, such a model describes the phenomenon of propagation of radio waves which attenuate with distance. In our study we consider the transmitter to be a transmitter communication tower and the receiver to be a mobile phone. The irregular lattice only is considered as this captures a realistic situation. Our irregular lattice distribution is created using a square grid covering an area of 2.2km×2.2km consisting of 484 smaller squares and each containing two transmitters deployed with a uniform distribution. Each transmitter covers a certain geographical area which is considered to be a cell in the irregular lattice.

The self-organisation frequency assignment rules adopted require some explanation. In principle neighbouring transmitters cannot use the same frequency. However the definition of neighbour is not simple since the interference depends not only on the distance between transmitters using the same frequencies, but also on the distance between the receiver and the interfering transmitter. Secondly as the performance of a receiver will be affected by the sum of the interference generated by multiple transmitters (we will refer to this as aggregate interference), the interference generated by any one transmitter is only considered harmful when other transmitters are interfering with the same receiver. In summary, whether any two transmitters are considered as neighbours depends on the distribution of the receivers and the frequencies on which the transmitters are operating at that time in question. For this reason, the decision on the frequency assignment is not based upon the neighbourhood relationship between transmitters, but is driven by the SINR experienced by the transmitters. As this metric takes into account the effect of the aggregate interference it is a good indicator of the presence of interference in the used channel. The simulation therefore also requires as inputs a receiver distribution and an SINR threshold below which interference is experienced.

Though in the longer-term vision each transmitter can use any spectrum that is accessible (as it gets it opportunistically, or through a spot auction or via a trade for example), in the simulation five 1 MHz channels in the 2 GHz band are used. It is worth re-emphasizing here that purpose of the simulation is to provide a framework for studying the complexity of the collective behaviour of the independent self-organising systems/transmitters rather than the specifics of the frequency assignment task. Our transmitters operate at transmit powers that are typical of modern 4G transmitters targeting local area coverage. The simulation parameters are summarized in Table 1.

The algorithm we propose for frequency assignment runs in a completely distributed manner as each transmitter makes a decision on which channel to use for the transmission in an autonomous way, only relying on local information with no inter-transmitter communication or signalling involved. The algorithm consists of two parts, the initialization phase and the channel selection phase. In the initialization phase, the algorithm picks one of the five available channels with equal probability. Then, in the channel selection phase, the transmitter will decide whether it is necessary to switch channel and, if so, which channel should be chosen. This operation is repeated periodically every N_{sw} time slots. The channel switching is triggered if the SINR of the worst receiver of the cell is below a given threshold $\mathrm{SINR}_{\mathrm{th}}$. In that case, the transmitter will choose the channel on which the majority of receivers experience an SINR higher than $\mathrm{SINR}_{\mathrm{th}}$. As mentioned earlier in this section, this algorithm runs in a self-organised and distributed manner since it only relies on the SINR data which are estimated by the receivers and then periodically fed-back to the transmitter. Every time a channel switching is triggered,

Table 1: Summary of the parameters used in the simulations.

System model	
Spectrum allocation	5 channels over 5MHz centered at 2 GHz. Bandwidth BW per channel is 1 MHz
Transmit power at transmitter	24 dBm (over 1MHz)
Transmitter antenna pattern/gain	Omnidirectional antenna / 0dBi
Receiver antenna pattern/gain	Omnidirectional antenna / 0dBi
Modeling of noise power P_N at the receiver	$P_{\mathrm{N}} = F \cdot \mathrm{BW} \cdot \mathrm{PSD_N}$, where $\mathrm{PSD_N}$ is the power spectral density of the thermal noise, F is the noise figure of the receiver.
Power spectral density $\mathrm{PSD_N}$ at the receiver	-174dBm/Hz.
Receiver noise figure	9 dB
SINR modeling	$\mathrm{SINR} = P_{\mathrm{RX}}/(I + P_{\mathrm{N}})$, where P_{RX} is the received power, I is the aggregate interference.
Scenario	
Transmitter position model	Square grid of 484 squares over an overall area of 2.2km×2.2km. 2 transmitters uniformly distributed over each square (968 transmitters overall)
Receiver position model	20000 Receivers uniformly distributed over the 2.2km×2.2km grid.
Propagation model	Signal power loss as a function PL(d) of the distance d, $\mathrm{PL}(d) = 140.7{+}36.7\log_{10}(d)$,[23].
Number of snapshots	100, with 100 different initial frequency allocations for each snapshot.
Algorithm parameters	
N_{ts}	5
N_{sw}	5
$\mathrm{SINR_{th}}$	4 dB

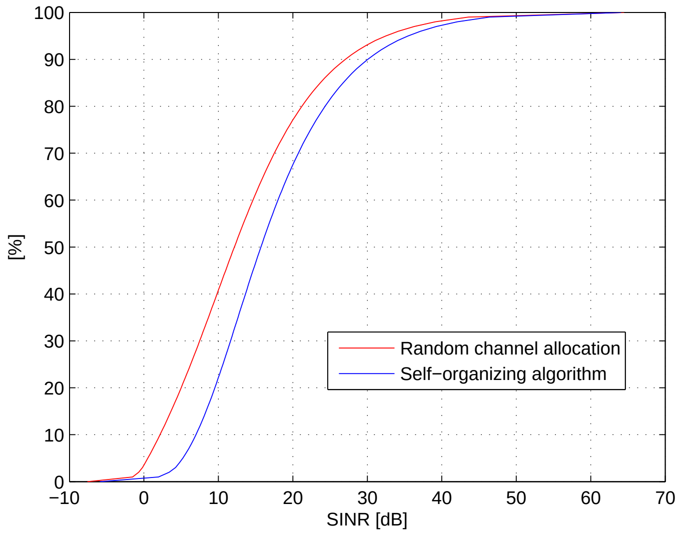


Figure 10: Comparison between random frequency allocation and proposed algorithm for the more realistic scenario.

the challenge becomes the creation of new communications link between the transmitter and its receivers. This can be done in multiple different ways. In conventional networks where channels are predetermined, this can be achieved through a process of beaconing and scanning. In case a predetermined channelization scheme is not in place, fixed frequency control channels could be used to disseminate the new carrier frequency. Alternatively, the transmitter can embed a cyclostationary signature in the waveform, for example using subcarrier mapping in case of multicarrier waveforms. Receivers can then use this signature to detect the signal of interest and estimate its parameters [24].

In order to avoid transmitters which are close to one another switching channel at the same time causing a "ping-pong" effect and likely leading to algorithm instability, we reduce the probability of neighbouring transmitters switching channel at the same time. In fact, in the initialization phase each transmitter randomly selects one out of N_{ts} time slots as a starting point for the channel selection phase; from this time slot onwards, the frequency selection operation will be performed periodically every N_{sw} time slots. This simple mechanism guarantees a low probability that neighbour transmitters switch channel at the same time. In addition, it does not require any inter-transmitter signalling for synchronization purpose.

The proposed frequency assignment algorithm shows good performance in terms of interference reduction and convergence. To analyse the behaviour of the algorithm, in Figure 10, we compare the SINR Cumulative Distribution Function (CDF) of the users for a random channel allocation (i.e., each transmitter selects one of the five available channels and does not deviate from this initial choice) with the proposed algorithm. The SINR CDF has been obtained by merging the receivers' SINR data collected across the network, for 100 different snapshots and for 100 different initial frequency allocations. From this plot we can see there is an enhancement of the SINR curve due to the reduced inter-transmitter interference. For example at the 10-percentile SINR, the proposed algorithm provides 5dB gain compared to a random frequency

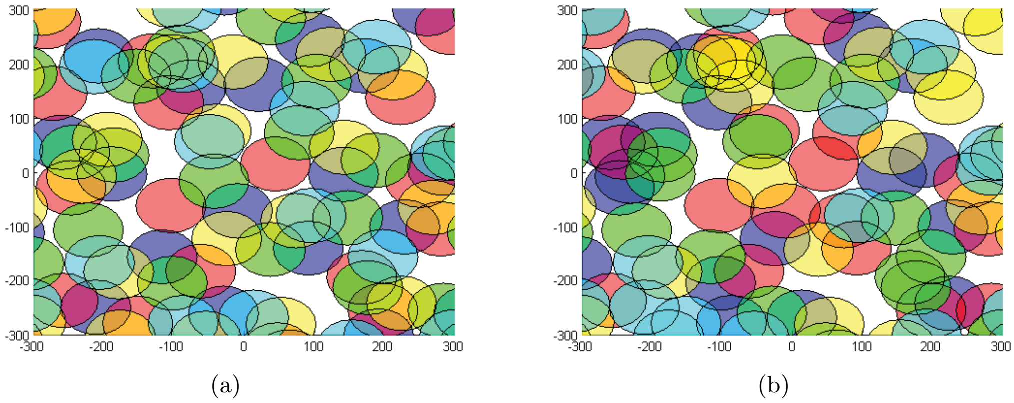


Figure 11: (a) Channel assignment resulting from the self-organising algorithm described in Section , $E_C = 0.55$ and $h = 2$. (b) Random channel assignment, $E_C = 0$ and $h = 2.32$.

allocation. In terms of convergence, we consider that the algorithm achieves a stable frequency allocation at channel X if, once the transmitter has switched to channel X (at time t_{X}), it will not switch to any other channel from time slot t_{X} onwards, regardless of whether the close transmitters keep switching their channel. From the simulation results it turns out that, on average, 97.34% of the transmitters achieve a stable frequency allocation in the scenarios we have tested, with $1.28 \cdot 10^{-4}$ variance.

It is worth reiterating at this point that the purpose of this section of the paper is not by any means to suggest that this self-organising frequency assignment algorithm is original or indeed an optimal self-organising algorithm but instead to provide one suitable self-organising approach that can be used as a basis for our analysis of the collective behaviour of such independent systems.

5.1 Complexity and Entropy for the Realistic Scenario

We can now study the global behaviour of our system from a complexity perspective. Figure 11(a) shows the frequency assigned to each transmitter of the irregular lattice using the frequency assignment algorithm described above based on the parameters in Table 1. Figure 11(b) shows the random assignment of frequencies. Different colors correspond to different frequencies. On visual inspection it is much harder to see structure. The cellular automata case gives a much clearer visual indication of the repetitive, complex and random nature of the behaviour exhibited by the collection of self-organising systems. However Figure 11 can only be properly understood in combination with the complexity metrics.

Hence we estimate E_C and h according to definitions given in Section for the two types of frequency assignments in Figure 11 using 2×10^5 cells. In the case of the channel assignment in Figure 11(a), the entropy estimates are $h(M) = 2$, $\forall M = \{4, 5, 6\}$. Hence, $h = 2$ and the resulting E_C is approximately 0.55. For the random channel assignment, the entropy estimates are $h(M) = 2.32$, $\forall M = \{1, 2, \ldots, 6\}$. Hence, $E_C = 0$ and $h = 2.32$. As the channel assignment is completely disordered, the entropy is the maximum possible for $N = 5$ channels. As previously observed for the case of the regular lattice discussed in Section , the channel assignment emerging from self-organisation exhibits a certain amount of structure which the random channel allocation cannot reach.

6 Impact of complexity on the stability of channel allocation

Now that we have gained some insights into the behaviour of the self-organising wireless nodes from a complexity perspective, the goal is to understand their implication in terms of channel allocation stability.

In this section we analyse the channel allocations discussed in Section with respect to their robustness to local changes in the environment. Let us denote by S_{CA} and S_{SOA} the channel allocation resulting from a centralized frequency planner and from the self-organising algorithm described in Section respectively. Let us assume that cell n in the lattice is required to change its frequency of operation from s_n to $\tilde{s}_n$. This would happen if a frequency which was previously available becomes no longer available. The network will have to find replacement frequencies (e.g. opportunistically or via trade). If we consider all possible cells n in the lattice and, for each cell, all possible channels $\tilde{s}_n \in \{\{1, 2, \ldots, N\} - \{s_n\}\}$, we obtain the sets $\tilde{\mathcal{S}}_{CA}$ and $\tilde{\mathcal{S}}_{SOA}$ of locally perturbed channel assignment matrices resulting from a centralized frequency planner and self-organised allocation respectively. In the case of an R-by-R lattice and N channels, the cardinality of each set is equal to $(N-1)R^2$. Each channel assignment $\tilde{S}_n \in \tilde{\mathcal{S}}_{CA}$ ($\tilde{S} \in \tilde{\mathcal{S}}_{SOA}$) differs from S_{CA} (S_{SOA}) only by the frequency used by cell n.

We define the *stability* of the channel allocation S_{CA} (or S_{SOA}) in terms of the minimum distance c between channel assignments in $\tilde{\mathcal{S}}_{CA}$ (or $\tilde{\mathcal{S}}_{SOA}$) and an interference-free channel allocation. The distance between two channel allocations is the number of changes that are necessary to *move* from one configuration to the other. In particular, we are interested in studying c when only cells in a neighbourhood $V_r(n)$ of radius r of the cell n, i.e. cells with coordinates (i, j) such that $\max([|i - i_n|, |j - j_n|]) \le r$, are allowed to change their channel.

Denote by x_i the N-dimensional binary vector such its j-th component x_{ij} is equal to 1 if and only if cell i uses the j-th frequency. For any $\tilde{S}_n \in \tilde{\mathcal{S}}_{CA}$ (or $\tilde{S}_n \in \tilde{\mathcal{S}}_{SOA}$), the closest interference-free channel allocation can be computed as the solution to the following semiquadratic integer problem:

$$
\begin{aligned}
&\min_{x_{ij}} \frac{1}{2} \sum_{i \in V_r} \sum_{j=1}^{N} \left(x_{ij} - x_{ij}^{(c)} \right)^2 \\
&\text{s.t.} \\
&\sum_{j=1}^{N} x_{ij} = 1, && \forall i \in V_r \\
&x_{ij} + x_{kj} \le 1, && \forall j \in \{1, 2, \ldots, N\}, \forall i \in V_r, \forall k \in V_1(i) \\
&x_{ij} = x_{ij}^{(c)}, && \forall j \in \{1, 2, \ldots, N\}, \forall i \in \{V_{r+1}(n) - (V_r(n) \cup \{n\})\} \\
&x_{ij} \in \{0, 1\}, && \forall j \in \{1, 2, \ldots, N\}, \forall i \in V_{r+1}(n)
\end{aligned}
\tag{3}
$$

where the N-dimensional binary vector $x_i^{(c)}$ denotes the channel used by cell i in the allocation $\tilde{S}_n$.

The first constraint in (3) ensures that all the cells obtain exactly one channel; the second constraint ensures that, for any cell i, none of the cells in its Moore neighbourhood ($V_1(i)$) can use the same channel as i; finally, only cells in the neighbourhood $V_r(n)$ of cell n - with the exclusion of cell n itself - are allowed to modify their channel. It should be noted that, in order to avoid interfering with the remaining cells in the lattice, cells in $V_{r+1}(n) - V_r(n)$ are included in the formulation in (3), but they are not allowed to modify their channel (see third constraint in (3)). In other words, the value of the variables that refer to those cells is fixed. For this reason, the first constraint and the objective function in (3) do not include

Table 2: Probability of minimum number of changes c.

Channel Allocation	Neighbourhood radius	$Prob$ (no solution)	$Prob$ $(c = 0)$	$Prob$ $(1 \leq c \leq 4)$	$Prob$ $(c \geq 5)$
Self-organised ($\tilde{\mathcal{S}}_{SOA}$)	$r = 1$ $r = 2$	0.21 0.0	0.06 0.06	0.67 0.86	0.06 0.08
Centralized planner ($\tilde{\mathcal{S}}_{CA}$)	$r = 1$ $r = 2$	0.4 0.0	0.0 0.0	0.4 0.6	0.2 0.4

cells in $V_{r+1}(n) - V_r(n)$. The minimum number of changes c that is required to obtain an interference-free allocation from $\tilde{S}_n$ is given by the objective function in (3).

Before proceeding, it is worth remarking that our aim is not to propose the formulation in (3) as a solution to the problem of determining an optimal interference free assignment of channels after a change has occurred. Our goal with the formulation in (3) is to analyse the channel allocation resulting from a centralized frequency planner and from the self-organising algorithm described in Section with respect to their stability. In order to compare the stability of the two types of channel allocation, we run 10^2 instances of the self-organising frequency allocation algorithm using $10^2 \times 10^2$ lattices. Then, for each resulting channel allocation, we consider all possible cells n and, for each cell, all possible frequencies, and we compute the optimal minimum distance to an interference-free channel allocation according to (3). In the case of centralized channel allocation, given the repetition of an identical basic pattern, only 6 different cell positions have to be considered. Table 2 shows the probability for a range of values of c resulting from these simulations. The locally perturbed channel allocation matrices resulting from self-organisation are more stable than the ones resulting from a centralized frequency planner. For example, in the case of $r = 1$, i.e. when only the 8 cells surrounding cell n are allowed to change their state, the probability that no solution to (3) exists in the case of centralized allocation is twice as large as in the case of self-organised allocation. Also, when $r = 2$, the $Prob(c \geq 5)$ is 0.08 for $\tilde{\mathcal{S}}_{SOA}$ and 0.4 for $\tilde{\mathcal{S}}_{CA}$. In other words, a locally perturbed channel allocation resulting from a central frequency planner requires a larger number of changes to return to an interference-free allocation, than in the case of self-organised channel assignments.

We also have tested the robustness of the self-organising algorithm in scenarios in which new transmitters and new receivers become active. This is because in future systems network infrastructure will be deployed by users without central planning. The experiments we run are carried out as follows. Given a stable frequency allocation obtained using the self-organising algorithm presented in Section for a scenario of N_{cell} cells, we add a new transmitter, again based on a uniform distribution, and we study whether this new transmitter perturbs the previous channel allocation achieved by the algorithm. We run the simulations for 10 different snapshots and, for each snapshot, 50 different initial frequency allocations with 5 channels are used. The simulation parameters we used are those listed in Table 1.

We would expect that, given the self-organising nature of the frequency allocation algorithm, the new transmitter is able to select a channel that provides satisfactory SINR values to its own receivers and, at the same time, only requires small changes of frequency in the neighbouring transmitters. In fact, the simulation results show that the self-organising algorithm in the new transmitter converges and is able to choose a channel such the interference is low enough to guarantee a good SINR for the users. The convergence percentage results in this experiment are in line with the results we obtained in Section . In addition, it turns out that less than 0.1% of the transmitters have to change channel due to the presence of the new transmitter.

7 Conclusions

The aim of the paper was to explore whether the behaviour of a collection of autonomous self-organising wireless networks can be treated as a complex system. The paper, more specifically, focused on self-organisation from a frequency perspective, based on emerging trends that see a move towards networks that access spectrum dynamically and a huge increase in small cells and user deployed infrastructure. For networks that allocate frequencies in a self-organised manner, we showed it does make sense to talk in terms of complex systems. In contrast to cases where frequency assignment is managed centrally and in which the resulting frequency allocation has a regular crystal-like configuration, these frequency allocations contain a certain amount of structure, not always obvious on a quick visual inspection but clearly present when complexity metrics are used in the analysis. Hence we claim that based on complex systems science definition, autonomous networks assigning frequencies to their cells in a self-organised way, can be defined and therefore studied, as *complex systems.* We recognize that our work is based on the use of specific self-organising algorithms as there is no one means of modelling all self-organising algorithms generically.

The fact the system is complex does have some advantages. In the examples we studied, we showed that the response to a change (because for example a frequency is no longer available and another might need to be used or because of the emergence of a new transmitter) happens in a more manageable fashion causing less disruption. This is a very welcome feature as it means the network is more robust to change. It is also possible to turn this idea on its head and make the suggestion that current cellular networks should refrain from central frequency planning and instead allow each macrocell (as well as pico and femtocell) to self-organise. The argument here is that in a world in which new transmitters are added to the system in an almost ad-hoc like fashion that the overall system would be more robust to this newer form of network evolution as distinct from the planned upgrades of the past.

We did not consider spectral efficiency in this paper. For example, in the case of the cellular automata model with a Moore neighbourhood, it is possible to use 4 frequencies in a centrally planned solution as distinct from the 6 needed in our self-planning system. Whether it is possible for a self-organising system to be as efficient has not been considered. However in analysing the robustness of the resulting frequency plans to change, 6 frequencies were used in the case of centrally and non-centrally planned systems to ensure the self-organising systems did not simply benefit due to the additional frequencies. It may be necessary in future work to consider spectral efficiency as well of course to consider more dynamic availability/lack of availability of spectrum. There is much more work which needs to be done. We plan as well to show whether the benefits of complex self-organizing systems, including robustness to changes in the environment (such as base stations switching on/off and spectrum being available or not on a highly dynamical fashion) tend to be increasingly more substantial if one moves towards densely deployed networks based on small basestations covering radii of few meters, which is one of the main trends foreseen for future mobile and broadband systems. Robustness will play a major role since the topology configuration of such networks is expected to be changing very dynamically and in a very granular fashion, leading to an increased risk of network instability. We intend to study whether results and tools from network science, dynamical systems and chaos theory can also be of use when studying issues related to capacity, coverage and topology configuration in the wireless networks we are interested in.

References

[1] Q. Zhao and B. M. Sadler, A survey of dynamic spectrum access, *Signal Processing Magazine, IEEE*, 2007, **24**(3): 79–89.

[2] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey, *Computer Networks*, 2006, **50**(13): 2127–2159.

[3] M. Mitchell, **Complexity - A Guided Tour**. Oxford University Press, 2011.

[4] J.-P. Onnela, J. Saramaki, J. Hyvonen, G. Szabo, D. Lazer, K. Kaski, J. Kertesz, and A.-L. Barabsi, Structure and tie strengths in mobile communication networks, in *Proceedings of the National Academy of Sciences*, May 2007, **104**(18): 7332 – 7336.

[5] C. A. Hidalgo and C. Rodriguez-Sickert, The dynamics of a mobile phone network, *Physica A: Statistical Mechanics and its Applications*, May 2008, **387**(12): 3017–3024.

[6] J. Candia, M. C. Gonzalez, P. Wang, T. Schoenharl, G. Madey, and A.-L. Barabsi, Uncovering individual and collective human dynamics from mobile phone records, *Journal of Physics A: Mathematical and Theoretical*, May 2008, **41**(22): 1–11.

[7] P. Wang, M. Gonzalez, C. A. Hidalgo, and A.-L. Barabsi, Understanding the spreading patterns of mobile phone viruses, *Science*, Mar. 2009, **324**(5930): 1071–1076.

[8] S. Lloyd, Measures of Complexity: A Nonexhaustive List, *IEEE Control Systems Magazine*, Aug. 2001, **21**(4): 7–8.

[9] D. Lopez-Perez, X. Chu, A. Vasilakos, and H. Claussen, On Distributed and Coordinated Resource Allocation for Interference Mitigation in Self-Organizing LTE Networks, *IEEE/ACM Transactions on Networking*, Aug. 2013, **21**(4): 1145–1158.

[10] F. Ahmed, O. Tirkkonen, M. Peltomäki, J.-M. Koljonen, C.-H. Yu, and M. Alava, Distributed Graph Coloring for Self-Organization in LTE Networks, *Hindawi Journal of Electrical and Computer Engineering*, 2010.

[11] J. Herzen, R. Merz, and P. Thiran, Distributed Spectrum Assignment for Home WLANs, in *IEEE INFOCOM*, 2013, 1573–1581.

[12] M. Putzke and C. Wietfeld, Self-Organizing Fractional Frequency Reuse for Femtocells Using Adaptive Frequency Hopping, in *IEEE Wireless Communications and Networking Conference (WCNC)*, 2013, 434–439.

[13] A. Stolyar and H. Viswanathan, Self-organizing Dynamic Fractional Frequency Reuse in OFDMA Systems, in *IEEE INFOCOM*, 2008, 1573–1581.

[14] I. Macaluso, L. Doyle, and L. DaSilva, Learning Nash equilibria in distributed channel selection for frequency-agile radios, in *ECAI Workshop on Artificial Intelligence for Telecommunications and Sensor Networks (WAITS)*, 2012.

[15] I. Macaluso, D. Finn, B. Ozgul, and L. DaSilva, Complexity of spectrum activity and benefits of reinforcement learning for dynamic channel selection, *IEEE Journal on Selected Areas in Communications*, 2013, **31**(11): 2237–2248.

[16] L. Gavrilovska, V. Atanasovski, I. Macaluso, and L. A. DaSilva, Learning and reasoning in cognitive radio networks, *IEEE Communications Surveys & Tutorials*, 2013, **15**(4): 1761–1777.

[17] D. P. Feldman and J. P. Crutchfield, Structural information in two-dimensional patterns: Entropy convergence and excess entropy, *Physical Review E*, 2003, **67**(5).

[18] R. Lopez-Ruiz, H. L. Mancini, and X. Calbet, A statistical measure of complexity, *Physics Letters A*, 1995, **209**(5-6), 321–326.

[19] H. V. Ribeiro, L. Zunino, E. K. Lenzi, P. A. Santoro, and R. S. Mendes, Complexity-entropy causality plane as a complexity measure for two-dimensional patterns, *PloS one*, 2012, **7**(8).

[20] Evolution Selects For and Against Complexity, *Lecture of Networks & Complex Systems, Indiana University*, 2007. [Online]. Available: http://vw.indiana.edu/talks-fall07/Yeager.pdf

[21] H. Beigy and M. Meybodi, A Self-Organizing Channel Assigment Algorithm: A Cellular Learning Automata Approach, Lecture Notes in Computer Science, 2003, **8**(3).

[22] ——, Cellular learning automata based dynamic channel assignment algorithms, *Journal of Computational Intelligence and Applications*, 2009.

[23] 3rd Generation Partnership Project (3GPP), Further Advancements for E-UTRA Physical Layer Aspects (Release 9), Mar. 2010, 3GPP TR 36.814 V9.0.0 (2010-03).

[24] P. D. Sutton, K. E. Nolan, and L. E. Doyle, Cyclostationary signatures in practical cognitive radio applications," *IEEE Journal on Selected Areas in Communications*, 2008, **26**(1), 13–24.