Cognitive Beamforming in Radar Bands

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Abstract—We investigate the problem of beamformer design for communications systems coexisting with legacy radar systems with mechanically rotating antennas. Based on measurement campaign findings that support the assumption of fast decoherence for the radar-to-base station interference within a radar rotation, as well as the assumption of very strong correlation of interference channel realizations corresponding to the same radar rotation phase, we introduce beamforming solutions exploring these channel characteristics, without imposing the need for current Channel State Information. Different beamformer designs are proposed, characterized by different levels of computational complexity, that are applicable both in the case of synchronous systems, where the communication system is aware of the radar rotation phase, as well as in the case of asynchronous systems.

Keywords—Cognitive Beamforming, Spectrum Sharing, Dynamic Spectrum Access, LTE, Radar

I. INTRODUCTION

The large blocks of spectrum currently allocated to radar services, e.g. radio-location, weather forecast, and radio-navigation, as well as their underutilization, has made radar bands a promising candidate for spectrum sharing [1]. While regulators have proposed the adoption of exclusion zones to achieve coexistence between radars and other communication systems [2], [3], such approaches can block a great percentage of communication devices, due to the large protection ranges required by radar systems. As a result, other more dynamic approaches, such as temporal sharing and cognitive beamforming, have also received considerable attention, e.g. [4], [5].

A particular challenge for spectrum sharing in radar bands is the fast decoherence of the interference channel, resulting from the fast rotation of the radar antenna, which may critically limit the effectiveness of these techniques. Briefly, we argue that the dynamics of the radio environment caused by reflecting objects and the constant motion of the radar antenna, in association with the intrinsic performance limitations of channel estimation techniques, may make it challenging for a Secondary User (SU) to keep track of the characteristics of the interference channel and, consequently, make the correct decision to transmit, selecting an appropriate precoder that ensures the protection of the Primary User (PU).

To overcome these limitations, in this work we present new, robust beamforming schemes for communication systems operating in radar bands. Our analysis is centered on the coexistence between Long Term Evolution (LTE) small cells and radar systems, due to the current interest shown by regulators and industry in deploying these systems in the 3.5 and 5 GHz bands [2]. We primarily focus on the LTE downlink channel, as the higher transmit power of evolved Node Base Stations (eNBs) compared to User Equipments (UEs) makes them the bottleneck in terms of exclusion zone sizes in radar bands [6], and investigate multi-antenna techniques at eNBs such as to achieve communication while respecting interference constraints introduced by radars. While the employment of multiple antenna techniques for spectrum sharing between radar and cellular systems has already been proposed in [7], [8], these papers focus on the shaping of the beamforming pattern at the radar side, a solution that may not be feasible for more rudimentary legacy single-antenna radar systems. The use of more futuristic MIMO radar systems and approaches like [9] based on radiolocation-communication co-design have the potential to significantly improve spectral efficiencies in radar bands, as they treat spectrum sharing as a cooperative problem. However, according to [10], single-antenna systems still constitute the majority, and, considering their high price and low cost of operation, replacing them for more advanced technologies may take several decades.

Differentiating our approach from existing schemes, in this paper we present beamforming solutions for LTE systems coexisting with legacy single-antenna mechanically-steering radars with limited interference mitigation capabilities. The proposed schemes account for both the case of systems that have knowledge of the operating characteristics of the radar, in particular, its instantaneous rotation phase, as well as for the case of systems having no knowledge of the operation characteristics of the radar. We illustrate the differences between the two cases, which we denote as synchronous and asynchronous beamforming schemes, respectively, in Figure 1. Neither scheme relies on instantaneous interference Channel State Information (CSI), but rather on CSI obtained in previous measurements of the interference channel. As we have shown in our RF measurement campaign [11], when both the radar and SU maintain fairly static positions, the interference channel displays cyclic characteristics across full revolutions of the radar antenna, and previous CSIs taken at specific radar antenna orientation angles are still valid when those angles are revisited in the following antenna sweeps. Hence, the cognitive beamforming schemes we propose can account for the effects of the fast decoherence time that characterizes the eNB-to-radar channel, which is difficult to accomplish using traditional methods for dealing with channel knowledge uncertainty, such as the ones presented in [12]–[15].

This paper builds upon our previous work in [16], proposing more advanced precoding techniques based on multi-objective optimization and Quadratically Constrained Quadratic Programming (QCQP). We also demonstrate the benefits of complementing cognitive beamforming with sharing techniques that rely on knowledge of the radar scan parameters. Finally,
we robustify the proposed techniques to account for the impact of channel estimation errors.

The main contributions of this work are summarized as follows:

- Based on our measurement campaign results, we analyze the impact of a radar antenna directivity and rotation on the interference channel. While, to the best of our knowledge, existing radar channel models focus on the radar reception itself, our model investigates the interference channel formed between the radar and multi-antenna communicating nodes. We then propose an adaptation of the existing WINNER channel model to better characterize this decoherence and its effect on beamforming schemes’ performance.

- By means of our proposed cognitive beamforming schemes, we tackle the problem of beamformer design for LTE/radar coexistence, assuming different levels of synchronization between eNB transmissions and radar rotation. We propose beamforming solutions, characterized by different levels of computational complexity, building on the theory of QCQP and Semidefinite Relaxation (SDR), as well on concepts of multiobjective optimization.

- We extend the proposed cognitive beamforming schemes to account for CSI uncertainties that may result from channel estimation errors or small-scale channel variations over time.

- We quantify through simulations the improvements brought by the proposed beamforming techniques in terms of exclusion zone size reduction, and small cells’ throughput increase.

We start with a quick mention of the notation adopted throughout this paper: Bold upper/lower case letters are used to denote matrices/vectors. The operator $\| \cdot \|$ stands for the Frobenius norm of a matrix/vector, while the operator $(\cdot)^T$ is used to denote matrix/vector transpose. Operator $\odot$ stands for the component-wise multiplication of vectors/matrices. Finally, we use notation $x \sim \mathcal{C}\mathcal{N}(0, R)$ to denote that random vector $x$ follows a complex multivariate Gaussian distribution with zero mean and covariance matrix $R$.

II. System Model

In Figure 2, we illustrate the sharing scenario under analysis. It comprises a radar system operating as PU, and a small cell eNB and $N_{\text{UE}}$ UEs as SUs. The radar system is equipped with a single directive antenna with a 3-D radiation pattern $G(\phi, \theta) \in \mathbb{R}$ that is used to transmit pulses and receive the resulting echoes in the azimuth and elevation angles $\phi$ and $\theta$, respectively. In order to scan the environment, the radar rotates its antenna orientation over time following a specific scan pattern $\Phi(t) = (\phi(t), \theta(t))$.

The secondary system comprises a Downlink (DL) LTE small cell eNB and the UEs which it serves. The secondary DL channel center frequency coincides with the radar’s frequency of operation. The eNB utilizes a half-wavelength spaced uniform linear dipole antenna array with $N_t$ elements. The eNB transmits a single stream per UE and serves one UE per Resource Block (RB). The UEs, in turn, are equipped with single dipole antennas.

For a single OFDM subcarrier, the symbols $s \in \mathbb{C}$ transmitted by the eNB reach the radar after being subject to the following transformation,

$$y = h_{\text{EIC}} w s + z. \quad (1)$$

Here $w \in \mathbb{C}^{N_t \times 1}$ is the eNB’s applied precoder, $z \in \mathbb{C}$ the AWGN at the radar’s receive antenna, and $h_{\text{EIC}} \in \mathbb{C}^{1 \times N_t}$ the Effective Interference Channel (EIC), which encompasses the effects of the propagation channel, hardware front-ends and antenna radiation patterns of the eNB and radar, and the current radar antenna’s orientation phase $\Phi(t)$. Considering the same set of antennas is utilized by the radar for transmission and reception, and by the eNB for DL transmission and channel estimation, and that both terminals perform prior calibration procedures, we can assume the reciprocity of the EIC.

Having defined the system model, in the remainder of the paper we will focus on the problem of beamforming for LTE systems coexisting with radars. Before presenting our proposed solutions, though, we first discuss how to model the channel formed between the eNB and the radar.

III. Channel Modeling Considerations

In order to characterize the channels formed between the eNB and the radar, and between the UEs and the radar,
as well as the impact that phenomena such as the radar antenna rotation and the presence of scatterers in the radio environment have on these interference channels, we have performed a channel measurement campaign in the Cork naval base, Ireland. Next, we summarize our findings during the campaign. Following that, we introduce the simulation model, employed throughout this work, that captures these findings.

A. Measurement Campaign in Cork Naval Base, Ireland

During our measurement campaign, we collected IQ samples of the transmissions of a Navy ship magnetron radar from multiple distances and propagation environments (both NLOS and LOS). The analyzed radar operates at the centre frequency of 3.05 GHz, with a transmit power of 30 kW, and its antenna full rotation (T^{ASP}), which we denote as Antenna Scan Period (ASP), takes approximately 2.6 seconds. We utilized a portable Tektronix RSA 306 spectrum analyzer with an instantaneous bandwidth of 40 MHz and an omni-directional antenna. More details on this measurement campaign can be found in [11].

In Fig. 3, we illustrate two examples of the time series of the radar Received Signal Strength (RSS), taken in NLOS and LOS conditions. Each dot represents a detected radar pulse. These measurements were obtained keeping both the radar and spectrum analyzer static. In LOS conditions, the RSS exhibits clear peaks at the time instants when the analyzer gets illuminated by the radar antenna main lobe. In contrast, in NLOS, the RSS is more distributed over time, indicating that the channel is not dominated by a strong direct propagation path, but by multiple reflection paths of comparable strength. These paths cause strong RSS peaks at different time delays, matching the moment when their associated scatterer gets illuminated by the radar antenna lobe. Considering the reciprocity of the interference channel, transmissions of the eNB should be carefully designed in order to ensure that the interference experienced by the radar is sufficiently small for the entire radar rotation period. This effect of reflections in the channel was also observed in the National Telecommunications and Information Administration (NTIA) measurements for the 3.5 GHz band [17].

In Fig. 4, we provide an illustrative example of the RSS autocorrelation of the observed radar’s transmit power in LOS for two different time scales. As can be seen in Fig. 4a, due to the fast intra-scan channel decoherence, the autocorrelation drops rapidly to below 0.5 for a lag higher than 10 ms. This interval is of the same order of magnitude as the time the magnetron radar antenna beam spends on a target, denoted as dwell time, of 14 ms. Fig. 4b, in turn, displays the strong periodicity of the RSS, causing RSS sample measurements separated by a duration interval of T^{ASP} = 2.6 seconds to be highly correlated. This effect, which we denote as inter-scan channel coherence, results from the fact that, when both the measurement equipment and radar positions are static, the interference channel physical features (e.g. scatterers’ positions and cross sections) remain fairly constant with time. As a result, channel samples measured by a receiver for a specific radar antenna rotation phase can be used as representative estimates of the channel for the following radar rotations for the same rotation phase.

B. Simulating the eNB-to-Radar channel

In this work, we employ the WINNER channel model as a basis to generate the channel realizations between the eNB and the radar system. As will become evident later in this section, this model captures several of the channel characteristics observed in our measurements campaign.

1) The WINNER channel model: The WINNER channel model is a geometry-based stochastic physical model that represents each Multiple Input Multiple Output (MIMO) channel impulse response matrix H between two terminals as a superposition of several direct and reflected propagation rays, each ray with a specific set of small scale parameters, namely delay, power, phase, Angle of Arrival (AoA) and Angle of Departure (AoD). A group of rays constitutes a cluster or propagation path of diffuse parameters, modelling the reflection of a single obstacle or scatterer in the environment. The resulting channel matrix can be represented by a linear combination of M_{path} propagation paths through the following expression,

$$
H(t; \tau) = \sum_{n=1}^{M_{\text{path}}} H_n(t; \tau) 
$$

where \( H_n(t; \tau) \) is the channel impulse response matrix of the propagation path \( n \) for the time instant \( t \) and a delay of \( \tau \). For the Tx and Rx antenna elements \( s \) and \( u \) respectively, the column \( s \) and row \( u \) element of \( H_n(t; \tau) \) can be defined as follows,

$$
H_{n,u,s}(t; \tau) = \sqrt{P_n} \sum_{m=1}^{M_p} F_{u}^{Rx}(\varphi_{n,m}) \Psi_{n,m} F_{s}^{Tx}(\phi_{n,m})
$$

where \( P_n \) is the transmit power of the path \( n \), \( M_p \) is the number of rays per cluster, and \( \varphi_{n,m}, \phi_{n,m}, v_{n,m}, \) and \( \tau_{n,m} \) the AoA, AoD, Doppler shift, and delay of the ray \( m \) for the path \( n \). The matrices \( F_{u}^{Rx} = [F_{u,V}^{Rx}, F_{u,H}^{Rx}]^{\top}, F_{s}^{Tx} = [F_{s,V}^{Tx}, F_{s,H}^{Tx}]^{\top} \) are field patterns of the Rx and Tx antenna elements \( u \) and \( s \), respectively in vertical and horizontal polarization, and \( \Psi_{n,m} \) models the channel polarization effect on each ray. Finally, \( \lambda_0 \) represents the signal wavelength, and \( d_u \) and \( d_s \) the distances between Rx and Tx antenna elements, respectively. For more details on this model, we suggest reading the document [18].

Each generated channel impulse response realization matrix \( \mathbf{H}^{EIC}(t; \tau) \in \mathbb{C}^{1 \times N_t} \), with time \( t \) and delay \( \tau \), can be equivalently represented in the frequency domain through \( \mathbf{h}^{EIC}[n, f] \in \mathbb{C}^{1 \times N_t} \), each corresponding to a specific RB’s time and frequency index \( n \in \mathbb{N} \) and \( f \in \mathbb{N} \) respectively. This one-tap conversion is valid as long as the EIC’s coherence bandwidth is not lower than the LTE RB’s bandwidth.

2) Extending the WINNER channel model: The generation of each ray’s small scale physical parameters, namely AoA, AoD, Doppler shift and attenuation, is carried out irrespective of the antenna radiation patterns of the terminal elements. This feature allows us to apply a posteriori the effect of the radar’s antenna field pattern \( F_{\text{radar}} \) and its rotation phase over time \( \Phi(t) \) a posteriori in the EIC impulse response matrix \( \mathbf{H}^{EIC}(t; \tau) \) generation through equation (3).
For the case when the radar and eNB maintain a fairly constant position, we can assume that the set of EIC propagation paths or clusters in the environment remains fairly constant over a full radar antenna rotation period of $T_{ASP}$. The main cause of the EIC’s decoherence within this period of time is, therefore, the sweeping motion of the radar antenna that we model through

$$F_{\text{radar}}(\Phi(t)) = \frac{1}{\sqrt{2}} \left[ \frac{\beta g(\Phi(t))}{(1 - \beta) g(\Phi(t))} \right].$$

(4)

Here $g(.)$ and $\beta$ are the linear antenna gain function and cross-polarization factor, respectively. The EIC impulse transfer matrix can then be fully defined through $H_{EIC}(\Phi(t); \tau) \in \mathbb{C}^{1 \times N_t}$. We model the eNB and UEs’ antennas as simple dipoles, and consider the channels from the radar and eNB to the UEs time-varying, as a consequence of the UEs’ movement.

As an illustrative example we simulated through the previously described WINNER channel model the received power by one of the eNB’s antenna elements from the radar for a full rotation period $T_{ASP}$. The resulting signal power per azimuth plot, which we denote as Perceived Antenna Radiation Pattern (PARP), together with the actual radar Antenna Radiation Pattern (ARP) at the transmitter, are displayed in Figure 5, after normalization by the main beam power. As can be seen, for some radar antenna orientation angles, the PARP and ARP are closely matched in amplitude. These portions correspond to the times when the EIC is dominated by the direct propagation path between the radar and the eNB. When channel clusters get illuminated by the radar main beam, strong spikes emerge in the PARP, similarly to the time domain spikes measured and illustrated in Figure 3. The angle difference between the main beam and these spikes’ positions coincides with the AoD of the respective cluster propagation path.

IV. COGNITIVE BEAMFORMING: CHALLENGES AND GOALS

The cognitive beamforming problem consists of finding the RB allocation and precoders that maximize the UEs’ throughput, while maintaining the instantaneous interference at the radar below a predefined threshold $I_{max}$:

$$\sum_{f=1}^{N_{\text{RB}}} \| h^{EIC}[f] w[f] \|^2 \leq I_{max},$$

(5)

where $N_{\text{RB}}$ is the number of LTE RBs, and $h^{EIC}[f]$ and $w[f]$ are the instantaneous EIC and selected precoder for the RB frequency index $f$. Multiple solutions already exist
### Intra-scan CSI staleness:
Our collected RF measurement data, illustrated in Section III, shows that a radar antenna’s very high directivity and rotation causes abrupt high magnitude range changes in the EIC within a radar rotation period. Thus, approaches like the ones presented in [12]–[15] are difficult to apply in our scenario, as they assume that the transmitter has a relatively up-to-date EIC estimate $\hat{h}^{\text{EIC}}$ and any estimation error can be bounded by an error region. The rapidly varying nature of EIC that we have observed in our measurements invalidates this assumption. For the rest of the paper we will refer to these rapid variations as **Intra-Scan Channel Decoherence**.

### Computational complexity:
Joint scheduling and precoding algorithms like the ones presented in [12]–[15] attempt to jointly treat the scheduling and precoder design problem, leading to complexity that makes it impractical to adopt those solutions in current LTE networks [19].

To overcome these two issues, we develop solutions to achieve the following goals:

**Robustness to Intra-Scan Channel Decoherence:** We propose beamforming schemes that do not rely on instantaneous EIC estimates, but rather on a set $S_{\text{EIC}}$ of EIC estimates obtained at time instances corresponding to different stages of the radar rotation cycle. In practice, this can be achieved if the eNB introduces channel estimation slots within data transmission slots, every $T_{\text{EIC,step}}$ seconds. Using such an approach, within a time interval $[0, T]$, the eNB obtains multiple EIC estimates $\hat{h}^{\text{EIC}}[n, f]$, where $n \in S_{\text{EIC}}$ is the estimation time index. Hence, $S_{\text{EIC}}$ can be defined as $S_{\text{EIC}} = \{0, \cdots, \left\lfloor \frac{T}{T_{\text{EIC,step}}} \right\rfloor - 1\}$. Clearly, the appropriate choice of parameter $T_{\text{EIC,step}}$ allows for collecting EIC estimates that correspond to different stages of the radar rotation cycle. This is accomplished if the period between estimates $T_{\text{EIC,step}}$ is lower than the radar antenna dwell time, and the time window of estimates $[0, T]$ is longer than the rotation period, i.e., $T > T_{\text{ASP}}$.

A question that arises when such an approach is followed is to what extent EIC estimates corresponding to a specific orientation of the radar at past time instances are representative of the current actual EIC, for the same orientation. In Section III, we have shown that when the radar and eNB maintain a relatively fixed position, the channel remains fairly unaltered for multiple radar antenna rotations. Thus, past EIC estimates can be used as estimates also for following radar antenna sweeps.

**Reduction of complexity:** To reduce the complexity of joint optimal scheduling and beamforming, we adopt sub-optimal approaches that decouple the two problems, providing in this manner a practical way to overcome the combinatorial nature of scheduling [19]. In particular, we combine existing, well established scheduling algorithms (e.g. Proportional Fair (PF)) with new precoder design algorithms.

Consider the coupled multi-UE and multi-RB throughput maximization objective function,

$$
\max_{w_u[f], u, f} \sum_{u=1}^{N_u} \sum_{f=1}^{N_f} \log \left(1 + \frac{\|\hat{h}_u^{\text{EIC}}[f]w_u[f]\|^2}{\sigma_0^2 + i_{\text{radar-UE}}} \right) \mu_{u,f}
$$

where $\mu_{u,f}$ is the binary decision to allocate user $u = 1, \cdots, N_u$ to RB $f = 1, \cdots, N_f$, $i_{\text{radar-UE}}$ is the interference caused by the radar on UE $u$ for the RB $f$, $\hat{h}_u^{\text{EIC}}[f]$ is the eNB to UE channel, $w_u[f]$ the selected precoder, and $\sigma_0^2$ the noise power. Following the proposed decoupling approach, the scheduling decision is performed by first finding the precoders that should be applied for each user/RB combination. That is, for every possible UE/RB association $u', f'$, we solve the beamforming problem, defined as:

$$
\arg \max_{w_{u'}[f']} \log \left(1 + \frac{\|\hat{h}_u^{\text{EIC}}[f']w_{u'}[f']\|^2}{\sigma_0^2 + i_{\text{radar-UE}}} \right)
$$

where $\hat{h}_u^{\text{EIC}}[f']$ is the normalized estimate for the channel between the eNB and the UE $u'$ on the $f'$ RB, which the eNB estimates based on the UE Precoding Matrix Index (PMI) feedback. Scheduling is then performed using a known scheduler that accounts for the rates achieved by the designed beamformer. Further details are given in the algorithmic description presented in Section IV-B.
A. The precoder design problem

Starting from (5) and (7), we define the DL precoder design for a specific UE $u$ and RB $f$ as

$$\begin{align*}
\text{maximize:} & \quad \|\hat{h}^{\text{UE}}_{u}[f]w_u[f]\|^2 \\
\text{subject to:} & \quad \|\hat{h}^{\text{EIC}}[n,f]w_u[f]\|^2 \leq i_{\text{max}}, \quad n \in \mathcal{E}_{\text{EIC}}, \quad (8) \\
& \quad \|w_u[f]\|^2 = P_{\text{eNB}},
\end{align*}$$

where $\hat{h}^{\text{UE}}_{u}[f]$ is the normalized eNB-UE channel estimate. Vectors $\hat{h}^{\text{EIC}}[n,f]$ are past EIC estimates forming a properly designed subset $\mathcal{E}_{\text{EIC}}$ of set $\mathcal{S}_{\text{EIC}}$. Finally, $i_{\text{max}} = \frac{i_{\text{max}}}{N_{\text{RB}}}$ is the threshold set on the interference caused at the investigated resource block. Essentially, the per resource block interference constraint introduced in (8) provides a way for decoupling the scheduling and beamforming problems. In more detail, notice that optimally solving the beamforming problem while satisfying constraint (5) requires a joint treatment of the scheduling and beamforming problems. In contrast, the per RB interference constraint introduced in (8) provides a way of decoupling the scheduling and beamforming problems, since beamformer design on each RB is now independent of the user that is scheduled on any other RB. For simplicity of exposition, from this point on we focus on a specific RB frequency and UE, and thus drop the indexes $f$ and $u$ in our notation.

Note that in (8) we have defined the eNB’s transmit power per RB, $P_{\text{eNB}}$, through an equality constraint rather than an inequality constraint, which is the approach followed in works [12]–[15]. We took this decision based on our particular focus on LTE. LTE as a standard does not provide the level of flexibility in the transmit power allocation that most cognitive radio works consider. While an operator can alter LTE stations’ DL transmit power according to its network topology, such configuration takes place at relatively long time scales. LTE intra-frame subcarrier-power allocation is more rigid, as this simplifies the eNB and UE hardware design and signaling schemes [20]. Dynamic power allocation becomes even more complex in spectrum sharing environments, where coexistence mechanisms (e.g. LBT) are employed. As a downside, this equality makes the solution $w$ to (8) suboptimal in terms of throughput and, in certain cases, infeasible. Nevertheless, as we discuss in Sections V-A and V-B, the beamformers that we propose in this work, can be easily extended to account for an inequality constraint at the transmit power $P_{\text{eNB}}$.

Finally, in what follows we present two different approaches for constructing $\mathcal{E}_{\text{EIC}}$, shown in Figure 1, which depend on the amount of information that we have about the radar.

1) The eNB has limited knowledge of operation characteristics of the radar: As a first case, we will assume that the eNB has only knowledge of the maximum possible $T_{\text{ASP}}$. Then, by selecting a set $\mathcal{E}_{\text{EIC}}$ containing the most recent channels in $\mathcal{S}_{\text{EIC}}$, such that $|\mathcal{E}_{\text{EIC}}| \times T_{\text{EIC,step}} \geq T_{\text{ASP}}$, the $\mathcal{E}_{\text{EIC}}$ encompasses all the obtained EIC estimates for at least one full radar rotation period $T_{\text{ASP}}$. Hence, the eNB will try to simultaneously respect the interference constraint $i_{\text{max}}$ for all the possible radar rotation phases. The advantage of this scheme is that it does not require the eNB to be aware of the current radar rotation phase. This constitutes a strong advantage, since in several cases, radars display non constant $T_{\text{ASP}}$, e.g. weather and tracking radar systems. We refer to the beamformer obtained under this assumption as an asynchronous beamformer.

2) The eNB has full knowledge of the operation characteristics of the radar: As a second case, we assume that the eNB knows or is able to estimate the radar antenna scan parameters, namely its rotation period $T_{\text{ASP}}$ and instantaneous rotation phase. The eNB can then select to populate $\mathcal{E}_{\text{EIC}}$ using channel estimates, selected such that

$$\|\Phi(nT_{\text{EIC,step}}) - \Phi_{\text{now}}\| < \gamma_{\text{EIC}},$$

where $\Phi_{\text{now}}$ is an estimate of the current radar rotation phase, and $\Phi(nT_{\text{EIC,step}})$ is an estimate of the rotation phase to which channel estimate $\hat{h}^{\text{EIC}}[n]$ corresponds. Finally, $\gamma_{\text{EIC}}$ is a threshold proportional to the degree of confidence the eNB has regarding the current radar rotation phase $\Phi_{\text{now}}$. Essentially, such an approach resembles the temporal sharing technique presented in [4], [21], [22], generalized to the multi-antenna case. We, therefore, refer to the beamformers derived based on the assumption of such knowledge as synchronous beamformers.

Assuming the knowledge of antenna scan parameters allows for reducing the cardinality of $\mathcal{E}_{\text{EIC}}$, and increase of the beamformer’s feasible region. Thus, the SNR that can be achieved is at least equal to the SNR achieved by the asynchronous beamformer. On the other hand, knowledge of the radar’s rotation period and instantaneous rotation phase may not be feasible in some particular scenarios, such as for some weather forecast and military systems [4].

B. Summary of the Algorithm

In what follows, we present the algorithm employed by an eNB in order to reach the final precoder and RB allocation solution for all its bandwidth of operation and UEs. The eNB, for each Transmission Time Interval (TTI), selects the subset of radar channel estimates to take into account $\mathcal{E}_{\text{EIC}}$, and receives through its feedback channel a list of UE-RB normalized channel estimates $\hat{h}^{\text{UE}}$ and Channel Quality Indicators (CQIs). Based on the CQIs, the eNB additionally obtains a quantized estimate of the UEs’ SNRs ($\text{SNR}_{u,f}$) before any precoding is applied. Then, for each RB $r$ and UE $u$, the eNB computes the precoder solution $w_u[f]$, the SNR corresponding to this precoder, that is denoted as $\text{SNR}_{u,w,f}$, and the corresponding throughput as given by Shannon’s formula. These values are then stored in the matrices $W$ and $C$, respectively, and used by the PF scheduler to get the appropriate UE-RB allocation.

V. Precoder Solutions

In this section, we describe the precoder computation algorithms that we have examined for maximizing the UEs’ throughput while avoiding excessive interference caused by the eNB to the radar. As a first approach we treat the beamformer design as a single-objective optimization problem where we
Algorithm 1 Algorithm for decoupled LTE precoding and scheduling

for each new TTI $t$ do
    select $B_{EIC}$ from $S_{EIC}$ at TTI $t$
    receive $h_{\text{UE}}^n$ feedback from all UEs and for all RBs
    receive CQIs from all UEs for all RBs
    compute pre-precoding SNR, $[f]$ for all CQIs
    $W \leftarrow \text{Matrix}(N_{\text{UE}}, N_{\text{RB}})/\text{matrix of computed precoders}$
    $C \leftarrow \text{Matrix}(N_{\text{UE}}, N_{\text{RB}})/\text{matrix of expected Shannon link capacities}$
    for each $f = 0, \cdots, N_{\text{RB}}$ do
        get $h_{\text{EIC}}[f]$ for all $n \in B_{\text{EIC}}$
        for each $u = 0, \cdots, N_{\text{UE}}$ do
            $w_u[f] \leftarrow \text{solve opt} (h_{\text{UE}}[n, f], h_{\text{EIC}}[f], i_{\text{max}}, P_{\text{SNR}})$
            $W[u, f] \leftarrow w_u[f]$
            // expected SNR with precoder is used as input to the scheduler
            $\text{SNR}_{u, w}[f] \leftarrow \text{SNR}_{u, f}[f] \cdot h_{\text{UE}}[w_u[f]]$
            $C[u, f] \leftarrow \log(1 + \text{SNR}_{u, w}[f])$
        end for
    end for
    $\text{UE} \_\text{alloc} \_\text{matrix} \leftarrow \text{PF}\_\text{scheduler}(W, C)$
end for

A. A throughput-maximizing approach

Initially, we attempt to calculate the beamformer $w$, by solving optimization problem (8). This is a QCQP. In their general form, QCQPs belong to the category of NP-hard problems [23]. Nevertheless, following a semi-definite relaxation approach, we can obtain approximate solutions to such problems, with performance bounds and polynomial complexity.

Following the relaxation steps presented in [23], we obtain an approximate solution $\tilde{w}$ through interior-point algorithms with a worst-case algorithmic complexity $O(\max\{|B_{\text{EIC}}|, N_t\}^4 \sqrt{N_t \log(1/\varepsilon)})$, where $\varepsilon > 0$ is the relaxed solution’s accuracy. For further details, we suggest consulting [23].

The approximate solution $\tilde{w}$, however, may not respect the original constraints of (8). To circumvent this issue, we map $\tilde{w}$ to a feasible solution $\tilde{w}$ by first normalizing it to the desired transmit power $W = \tilde{w} \sqrt{P_{\text{SNR}}/\|W\|^2}$. For most of the cases, $W$, after this scaling step, respects the interference constraints, and we can set it as the final relaxed QCQP solution. Otherwise, we repeat the computation of $\tilde{w}$, this time with more strict interference constraints. The updating of these constraints is carried out as follows:

\[
\{i_{\text{max}}\}_j = \{i_{\text{max}}\}_{j-1} - \alpha \left( \max_{n \in B_{\text{EIC}}} \|h_{\text{UE}}[n]w\|^2 \right),
\]

where $j \in \mathbb{N}$ is the recomputation step index starting at 1, $\{i_{\text{max}}\}_j$ is the maximum interference constraint at step $j$, $i_{\text{max}}_0 = i_{\text{max}}$, and $\alpha$ is a positive coefficient. As can be seen, the expression within brackets in (10) corresponds to the difference between the interference caused by the precoder solution obtained for the relaxed QCQP and the stipulated interference limit, which when infeasible, is always positive. The limit $\{i_{\text{max}}\}_j$ will decrease with each iteration by a factor proportional to this difference and the coefficient $\alpha$. The iteration will stop once a feasible solution is found. If we pick an $\alpha$ that is too small, we incur a higher chance to recalculate the solution to the semi-relaxed QCQP problem. If too high, the obtained solution may be suboptimal. For our simulations, we noticed that a feasible solution was generally found for the first QCQP calculation ($j = 0$). When not found, an $\alpha$ equal to 2 would generally lead to between 1 and 2 extra QCQP recalculations, which has a limited impact on the final complexity. As a final comment, we highlight that substituting the equality constraint on the transmit power by an inequality constraint, the resulting problem is still a QCQP and can be solved again by employing a semi-definite relaxation approach, similar to the one described above.

B. A multi-objective optimization approach

For an asynchronous beamformer, $|B_{\text{EIC}}|$ takes on very large values, significantly increasing the complexity of the throughput maximizing approach described in the previous section. As a lower complexity alternative, we propose the following multi-objective optimization problem:

\[
\begin{align*}
\text{maximize:} & \quad \|h_{\text{UE}}w\|^2 \\
\text{minimize:} & \quad \|h_{\text{EIC}}[n]w\|^2 \leq i_{\text{max}}, n \in B_{\text{EIC}} \\
\text{subject to:} & \quad \|w\|^2 = P_{\text{SNR}}.
\end{align*}
\]

After noticing that $\|h_{\text{UE}}w\|^2 = 1$ and, consequently, $\|h_{\text{UE}}w\|^2 \leq P_{\text{SNR}}$, (11), and applying linear scalarization, we can transform (11) to the following single objective optimization problem:

\[
\begin{align*}
\text{minimize:} & \quad \sum_{i=1}^{\lfloor B_{\text{EIC}} \rfloor} \mu_i \left( \|h_{\text{EIC}}[n]w\|^2 + \mu_i \|h_{\text{EIC}}[n]w\|^2 \right) \\
\text{subject to:} & \quad \|w\|^2 = P_{\text{SNR}}B, \\
\end{align*}
\]

where $\mu_i$, $i = 1, \cdots, \lfloor B_{\text{EIC}} \rfloor$ are the weights given to each interference constraint and $\mu_i \lfloor B_{\text{EIC}} \rfloor + 1$ to the UE’s throughput term. Moreover, we assume that weights $\mu_i$ follow the constraints:

\[
\sum_{i=1}^{\lfloor B_{\text{EIC}} \rfloor + 1} \mu_i = 1, \quad \mu_i \geq 0, i = 1, \ldots, \lfloor B_{\text{EIC}} \rfloor + 1.
\]
approach are then only deemed valid if they respect the original interference constraint (5).

One aspect not addressed in this multiobjective optimization transformation is the selection of the weights $\mu_i$ for each of the $|B_{EIC}|+1$ constraints in (12). One simple approach is to set $\mu_i$ equal for all constraints ($\mu_i = \frac{1}{|B_{EIC}|+1}, i = 1, \ldots, |B_{EIC}|+1$). However, as it will be shown in the results section, the use of varying values for coefficients $\mu_i$ can provide superior performance. Hence, in what follows we extend this beamformer design such as to enable assigning different values to coefficients $\mu_i$. In more detail, we design an algorithm that allows for the weight $\mu_{|B_{EIC}|+1}$, which is associated with the UE’s throughput, to have a different value as compared to the remaining weights, which are restricted to be the same. As a result, with this approach we can control the importance of rate maximization in the multiobjective optimization formulation of the beamforming problem. We further explain this approach in the following analysis.

As an extension of the QCLS beamformer, we consider using a sequential procedure that is based on deriving QCLS beamformers for different values of the coefficient $\mu_{|B_{EIC}|+1}$, using a binary search method, until an appropriate solution is found. This approach enables a finer control over the throughput-complexity tradeoff.

Our algorithm requires, as input, a list of all the possible candidate values $\{\mu_{|B_{EIC}|+1}^{(1)}, \ldots, \mu_{|B_{EIC}|+1}^{(K)}\}$ for UE’s weight $\mu_{|B_{EIC}|+1}$ can take. At each stage a $k = 1, \ldots, K$ is selected that defines $\mu_{|B_{EIC}|+1}^{(k)}$. The remaining weights $\mu_i^{(k)}$, $i = 1, \ldots, |B_{EIC}|$, that are associated with the interference constraints, are determined through $\mu_i^{(k)} = \frac{1-\mu_{|B_{EIC}|+1}^{(k)}}{|B_{EIC}|}$, $i = 1, \ldots, |B_{EIC}|$. The QCLS problem is then solved following the steps described in the previous subsection. The exact operation of this scheme is presented in Algorithm 2.

**Algorithm 2** Binary search algorithm to find the optimum $\mu_{|B_{EIC}|+1}^{(k)}$, $k = 1, \ldots, K$

\[
x_{L} \leftarrow \text{QCLS}(\mu_{|B_{EIC}|+1}^{(1)}) \quad \text{// Compute QCLS for the maximum } \mu_{|B_{EIC}|+1}^{(1)}
\]

if \( L_{R} \) respects interference constraints then
\[
x_{L} \leftarrow \text{QCLS}(\mu_{|B_{EIC}|+1}^{(1)}) \quad \text{// Compute QCLS for the minimum } \mu_{|B_{EIC}|+1}^{(1)}
\]
end if

// $x_{L}$ does not respect interference constraints then
\[
x_{L} \leftarrow \text{QCLS}(\mu_{|B_{EIC}|+1}^{(1)}) \quad \text{// Compute QCLS for the minimum } \mu_{|B_{EIC}|+1}^{(1)}
\]
end if

$R \leftarrow K$ \quad \text{// R points to the minimum } $\mu_{|B_{EIC}|+1}^{(1)}$
$L \leftarrow 1$ \quad \text{// L points to the maximum } $\mu_{|B_{EIC}|+1}^{(1)}$

while $L + 1 \leq R$ do
\[
M \leftarrow \frac{L + R}{2} \quad \text{// We cut the interval } [L,R] \text{ in half}
\]
\[
x_M \leftarrow \text{QCLS}(\mu_{|B_{EIC}|+1}^{(L+M)}) \quad \text{// Compute QCLS for the middle of } [L,R]
\]
if $x_M$ respects interference constraints then
\[
x_{L} \leftarrow M \quad \text{// Interval is shrunk from the right}
\]
end if
\[
x_{R} \leftarrow M \quad \text{// Interval is shrunk from the left}
\]
end while

The while loop of Algorithm 2 takes $\log_2(K)$ QCLS computations, making the complexity of the algorithm $O(\log_2(K)N^2)$. Considering that $N$ is in general small for non-massive MIMO systems, this approach is considerably less complex than the throughput maximization one. Finally, we note that in case that the equality constraint on the transmitted power in (12) is substituted by an inequality constraint, the resulting problem is again a QCQP with one inequality constraint and can be solved using standard convex optimization techniques. Having presented our precoder selection techniques, we will next focus on robustifying these techniques against channel estimation uncertainties.

**VI. CHANNEL ESTIMATION IMPRECISION**

The proposed beamforming solutions assume ideal channel estimation and no inter-scan channel decoherence, so that previous eNB estimates $\hat{\mathbf{H}}_{EIC}$ in $S_{EIC}$ remain valid in the following radar antenna rotation periods. In this section, we focus on scenarios where this assumption does not hold, and channel uncertainty can be modeled by a known statistical distribution. We extend the previous beamforming solutions to account for such uncertainty and ensure that the probability of causing interference above $I_{\text{max}}$ to the radar remains below a pre-defined value.

Similarly to [25], [26], we consider that actual channel realizations $\mathbf{h}_{EIC}$ are related to estimates $\hat{\mathbf{h}}_{EIC}$ through the following expression,

$$\mathbf{h}_{EIC}[f] = \hat{\mathbf{h}}_{EIC}[f] + \mathbf{h}'[f],$$

where $\mathbf{h}'$ is the channel estimation error that follows a known a priori statistical distribution. In what follows we show how knowledge of the statistical distribution of $\mathbf{h}'$, and more particularly knowledge of the statistical distribution of the squared norm of the channel error, defined as

$$\epsilon[f] = \|\mathbf{h}'[f]\|^2,$$

can be used to obtain robust beamformer designs. To this end, we assume that the eNB aims at selecting a beamformer such as to ensure that with probability $\mathcal{P}$, the interference constraint is satisfied, or equivalently:

$$P_{\mathcal{F}_c} \left( \sum_{f=1}^{N_{RB}} \left\| (\hat{\mathbf{h}}_{EIC}[n,f] + \mathbf{h}'[n,f]) \mathbf{w}[n,f] \right\|^2 \leq I_{\text{max}} \right) \leq \mathcal{P}. \quad (16)$$

The following theorem presents a technique for satisfying this constraint, provided knowledge of the CDF $\mathcal{F}_c(\cdot)$ of $\sum_{f=1}^{N_{RB}} \epsilon[f]$.

**A. Generalization to robust beamforming**

In case of partial CSI, the solution to (8) may exceed the original interference limit set by the radar operator. We address this issue in this subsection by generalizing our optimization problem (8) to the robust beamforming scenario, following a probabilistic approach.
Theorem 1: A beamformer satisfying the constraint (16) can be obtained by solving the following optimization problem

\[
\begin{align*}
\text{maximize:} & \quad \|\hat{h}^{\text{UE}} w\|^2 \\
\text{subject to:} & \quad \left( \sum_{n=1}^{N_{\text{RB}}} \left| \hat{h}_{\text{EIC}}^{\text{UE}}[n] w[n] \right|^2 \right) \leq \left( \sqrt{T_{\text{max}} - \frac{F^{-1}_{\epsilon}(P) P_{\text{eNB}}}{N_{\text{RB}}} } \right)^2, \quad n \in B_{\text{EIC}}, \\
\|w\|^2 & = P_{\text{eNB}}.
\end{align*}
\]

(17)

Proof: Noticing that \( \|\hat{h}^{\text{UE}} w\|^2 \leq \epsilon P_{\text{eNB}} \) and applying the triangle inequality followed by the Cauchy–Schwarz inequality, we can rewrite the interference constraint (16) as in (18) where the time index \( n \) was omitted for brevity. It is easy to see that inequality (18) can be satisfied provided that

\[
0 \leq \sum_{f=1}^{N_{\text{RB}}} \epsilon [f] \leq \frac{P_{\text{eNB}}}{\lambda} \left( \sqrt{T_{\text{max}} - \sum_{f=1}^{N_{\text{RB}}} \left| \hat{h}_{\text{EIC}}^{\text{UE}}[f] w[f] \right|^2 } \right)^2.
\]

(19)

Therefore, we can ensure that the probabilistic constraint is satisfied, provided that

\[
F_{\epsilon}(A) \leq P.
\]

(20)

Condition (20) can then be rewritten as follows,

\[
\frac{1}{N_{\text{RB}}} \sum_{f=1}^{N_{\text{RB}}} \left| \hat{h}_{\text{EIC}}^{\text{UE}}[f] w[f] \right|^2 \leq \left( \sqrt{T_{\text{max}} - \frac{F^{-1}_{\epsilon}(P) P_{\text{eNB}}}{N_{\text{RB}}} } \right)^2.
\]

(21)

Recalling our original practical goal of making the precoder selection invariant with the frequency, we convert (21) to the more restrictive condition,

\[
\|\hat{h}_{\text{EIC}}^{\text{UE}} w\|^2 \leq \left( \sqrt{T_{\text{max}} - \frac{F^{-1}_{\epsilon}(P) P_{\text{eNB}}}{N_{\text{RB}}} } \right)^2.
\]

(22)

That completes the proof.

From (17), we can conclude that the generalization of (8) to the robust beamforming case consists of adding an extra constant term to the interference constraint and does not have any impact on the problem’s algorithmic complexity. We also conclude from (17) that it is not possible for the eNB to transmit without causing interference above \( i_{\text{max}} \) in a specific RB with a probability higher than \( P \) if the right side of the interference constraint is negative, i.e. if the following condition is not respected,

\[
F_{\epsilon}(P) \leq \frac{i_{\text{max}} N_{\text{RB}}}{P_{\text{eNB}}} = \frac{I_{\text{max}}}{P_{\text{eNB}}}.
\]

(23)

B. Relation to the estimation error at the eNB

It is important to notice that the eNB is not able to estimate \( h^{\text{EIC}} \), but a scaled version \( \hat{h}^{\text{EIC},\text{eNB}} = h^{\text{EIC}} \sqrt{P_{\text{radar,Tx}}} \), proportional to the transmit power of the radar. Hence, finding the solution to (17) requires that the eNB be aware of the \( P_{\text{radar,Tx}} \) value, which could, for example, be provided by a database/Spectrum Access System (SAS). As we will show next, the \( P_{\text{radar,Tx}} \) scaling term also has important implications on the magnitudes of the error norm \( \epsilon \) that are expected for (17).

Consider that every channel estimate obtained by the eNB \( \hat{h}^{\text{EIC},\text{eNB}} \) contains an associated error \( e_{\text{eNB}} = \|h^{\text{eNB}}\|^2 \). Being aware of the radar transmit power \( P_{\text{radar,Tx}} \), the eNB obtains the term \( \hat{h}^{\text{EIC}} \) of the problem (17), by scaling its estimate \( \hat{h}^{\text{EIC},\text{eNB}} \). The end result of this procedure can be represented as follows:

\[
\hat{h}^{\text{EIC}} = \hat{h}^{\text{EIC},\text{eNB}} \sqrt{P_{\text{radar,Tx}}} = \frac{h^{\text{EIC},\text{eNB}} - h^{\text{eNB}}}{\sqrt{P_{\text{radar,Tx}}}} = h^{\text{EIC}} - h^{\epsilon}.
\]

(24)

As we can see from the previous equation, the channel uncertainty term \( \epsilon \) in (17) also differs from the eNB’s estimation error \( e_{\text{eNB}} \) by scaling factor \( P_{\text{radar,Tx}} \). Bearing this in mind, we represent our robust optimization problem (17) as a function of \( e_{\text{eNB}} \) as follows:

\[
\begin{align*}
\text{maximize:} & \quad \|\hat{h}^{\text{UE}} w\|^2 \\
\text{subject to:} & \quad \left( \sum_{n=1}^{N_{\text{RB}}} \left| \hat{h}_{\text{EIC}}^{\text{UE}}[n] w[n] \right|^2 \right) \leq \left( \sqrt{T_{\text{max}} - \frac{F^{-1}_{\epsilon}(P) P_{\text{eNB}}}{N_{\text{RB}}} } \right)^2, \quad n \in B_{\text{EIC}}, \\
\|w\|^2 & = P_{\text{eNB}}.
\end{align*}
\]

(25)

where \( F_{\epsilon}(\cdot) \) is the CDF of \( \sum_{f=1}^{N_{\text{RB}}} e_{\text{eNB}}[f] \). The transmission condition, in turn, is defined by

\[
I_{\text{max}} \geq \frac{P_{\text{eNB}}}{P_{\text{radar,Tx}}} F^{-1}_{\epsilon}(P).
\]

(26)

Taking into account that in general \( P_{\text{eNB}} \ll P_{\text{radar,Tx}} \), we expect that the previous condition is easily respected, and the eNB will be able to transmit without causing interference even when the channel uncertainty norm is above the noise floor.

VII. SIMULATION ANALYSIS

In this section, we assess the performance of our proposed beamforming techniques in terms of radar exclusion zone size reduction and impact on an opportunistic small cell’s throughput. Our results were obtained using the Matlab-based Vienna LTE System-Level simulator [27], modified to enable the modelling of the radar’s operation and interference effects.
\[
N_{\text{RB}} \sum_{f=1}^{N_{\text{RB}}} \left| \left( \mathbf{h}_{\text{EIC}}^* f \mathbf{w}_f \right) \right|^2 \leq N_{\text{RB}} \sum_{f=1}^{N_{\text{RB}}} \left| \mathbf{h}_{\text{EIC}}^* [f] \mathbf{w}_f \right|^2 + \sum_{f=1}^{N_{\text{RB}}} \epsilon_f P_{\text{ENB}} + 2 \sum_{f=1}^{N_{\text{RB}}} \left| \mathbf{h}_{\text{EIC}}^* [f] \mathbf{w}_f \right|^2 \cdot \sum_{f=1}^{N_{\text{RB}}} \epsilon_f P_{\text{ENB}} \leq I_{\text{max}}, \quad (18)
\]

A. Simulation scenario

The main parameters of the propagation model we adopted and technical specifications of the small cell are summarized in Table I. To model the pathloss within the small cell we use the Dual Slope model, which applies one pathloss exponent up to a certain distance from the small cell and another beyond it. These two pathloss exponents are taken from the 3GPP dense HeNB (femtocell) modelling guidelines, namely cases 3 (indoor) and 4 (outdoor) from [28, table 6].

The radar system’s parameters, in turn, are inspired by the ones of the magnetron radar analyzed in the NTIA measurement report [29] and are also summarized in Table I. Both the pathloss and fading model employed for the EIC are characterized in section III. The radar antenna only rotates in azimuth, and its Interference to Noise Ratio (INR) limit of -6 dB was obtained from the NTIA report [6]. Radar systems tend to have narrower bandwidths than LTE systems. To emulate this feature, we applied a spectrum mask to our radar transmit and receive filters that approximates the one displayed in the NTIA measurement report [17, Fig. 5]. Concerning the radar’s antenna radiation pattern, we employed the one suggested by the ITU-R in [3], which, for an antenna gain of 40 dBi, has a -3 dB beamwidth of around 1°.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmit Power (\text{dBm})</td>
<td>850 kW</td>
</tr>
<tr>
<td>Antenna Gain (\text{dBi})</td>
<td>40 dBi</td>
</tr>
<tr>
<td>Rotation Period (\text{s})</td>
<td>3 s</td>
</tr>
<tr>
<td>Centre Frequency (\text{GHz})</td>
<td>3.5 GHz</td>
</tr>
<tr>
<td>Maximum acceptable INR (\text{dB})</td>
<td>-6 dB</td>
</tr>
<tr>
<td>Centre Frequency (\text{GHz})</td>
<td>3.5 GHz</td>
</tr>
<tr>
<td>Bandwidth of operation</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Region of Interest (ROI)</td>
<td>100 m × 100 m</td>
</tr>
<tr>
<td>Propagation Model</td>
<td>Dual Slope</td>
</tr>
<tr>
<td>Indoor Region Radius</td>
<td>20 m</td>
</tr>
<tr>
<td>Location</td>
<td>Centre of ROI</td>
</tr>
<tr>
<td>Antenna Array</td>
<td>Uniform Linear Array</td>
</tr>
<tr>
<td>Estimation time step (\text{ms})</td>
<td>3.3 ms</td>
</tr>
<tr>
<td>Noise Floor (\text{dBm})</td>
<td>-107 dBm</td>
</tr>
<tr>
<td>UE Distribution</td>
<td>Uniform within ROI</td>
</tr>
<tr>
<td>Number of Antennas (\text{N})</td>
<td>1</td>
</tr>
<tr>
<td>Antenna Array</td>
<td>Cross-polarized</td>
</tr>
</tbody>
</table>

TABLE I: Simulation Specifications

We consider that at the beginning of each simulation run, the eNB has already \( |S_{\text{EIC}}| = 360 \) channel estimates. When employing synchronous beamforming schemes, the eNB at any instant of time \( t \) knows the current radar antenna orientation angle \( \Phi(t) \), with an uncertainty bound \( \gamma_{\text{EIC}} \) of 1°. The \( B_{\text{EIC}} \) is then computed as described in (9). On the other hand, for asynchronous beamforming schemes the eNB utilizes a \( B_{\text{EIC}} \) equal to \( 360 \). Each run can be characterized by a specific parameter configuration, UEs and EIC’s scatterers placement, and takes a simulation time equal to the primary radar system’s ASP of 3 seconds, which is equivalent to 3000 LTE TTIs.

B. Tested Precoding Schemes

In Table II, we describe the beamforming schemes we tested during our simulations, including two baseline schemes as well as schemes derived using our proposed precoding beamforming algorithms. As baseline schemes we have considered the asynchronous and synchronous MRT. The asynchronous MRT is the simplest scheme, as it ignores the radar antenna scan pattern, and only accounts for the \( t_{\text{max}} \) limit to define its precoder solution validity after the precoder computation. This scheme can be alternatively interpreted as a threshold-based spatial sharing approach similar to Dynamic Frequency Selection (DFS) used in the 5 GHz band [3].

Synchronous Maximum Ratio Transmission (MRT) can be interpreted as a simple temporal sharing scheme like [4, 21], as when employing it, the eNB will only resort to transmission interruptions to prevent the interference it causes if exceeding \( t_{\text{max}} \).

As for the proposed precoders, we test the asynchronous QCLS, multi-staged QCLS, and QCQP, and synchronous multi-staged QCLS. For the asynchronous QCLS we set equal weights \( \mu_i = \frac{1}{|B_{\text{EIC}}|+1}, i = 1, \ldots , |B_{\text{EIC}}| + 1 \). The performance of the asynchronous multi-staged QCLS algorithm depends on the choice of the list of weights \( \mu_i |B_{\text{EIC}}| + 1, k = 1 \ldots K \). Defining the throughput-interference weight coefficient \( g = \frac{1}{P_{\text{dB,EIC}}|B_{\text{EIC}}| + 1} \), we employed for the multi-staged QCLS algorithm the list \( \log_2 g \in \{+\infty, 18, 16, 14, 12, 11, 10, 9, 8, 7, 6, 4, 2, 0, -4, -\infty \} \), whose length is equal to \( K = 16 \). This corresponds to 6 QCLS computations per precoder solution. The first position on the list (\( q = +\infty \)) corresponds to the MRT case. The synchronous multi-staged QCLS is the only simulated scheme that combines the knowledge of the radar antenna rotation phase with an interference-aware precoding scheme.

Finally, we mention that the solutions obtained following our approach, are not always feasible solutions for the optimization problem initially introduced in (8). In fact, it is not always guaranteed that a precoding solution exists for a given RB and UE pair that simultaneously respects the
maximum interference constraint \( i_{\text{max}} \) (INR of -6 dB) of the radar, while keeping the eNB's transmit power constant. When such scenario occurs, we stipulate that the eNB forbids the PF scheduler to allocate the UE to the respective RB. For the particular case when none of the cell's UEs can be allocated to a given RB, the RB is left empty.

C. Throughput for one UE per small cell

Our first test consists of measuring the total small cell’s throughput and radar exclusion zone gains for one UE per cell, in a no estimation noise scenario, i.e. \( \hat{h}_{\text{EIC}} \). Figure 6a shows the average cell throughput as a function of distance to the radar. As can be seen, \textit{synchronous multi-staged QCLS} has the highest performance of all the schemes, providing an acceptable throughput even for distances of 500 meters from the radar. In contrast, asynchronous MRT has very poor performance, impacting the cell throughput even for distances beyond 60 km. This can be explained by the fact that under this scheme the majority of the DL RBs do not get allocated, to prevent the interference on the radar from exceeding the \( i_{\text{max}} \) limit. Synchronous MRT provides a significant improvement in relation to the asynchronous MRT. However, its performance decays faster than for any of the remaining asynchronous beamforming schemes. QCQP provides the best performance of all the four asynchronous beamformers. This demonstrates the quality of the semi-definite relaxation step we took to obtain approximate solutions to the original QCQP (8). If the complexity of this algorithm proves to be prohibitively high for small cell systems, the eNB should opt for the multi-staged QCLS instead, which provides significant improvements compared to its original QCLS form, at the cost of an extra logarithmic multiplicative term on its \( \mathcal{O}(\cdot) \) complexity.

While the results in Figure 6a are promising for the proposed beamforming schemes, one should not forget that cognitive beamforming is very sensitive to the positioning of the UEs. In particular, if a UE and radar channel are closely aligned, the eNB may not be able to find a precoder that provides acceptable Signal to Noise Ratios (SNRs) to UEs while respecting interference constraints. To evaluate the severity of this phenomenon, we computed the 0.5% and 99.5% quantiles of the UE’s throughput across all the performed simulation runs for each distance, which are shown in Figures 6b and 6c, respectively.

In Figure 6b, we observe that in poor channel conditions, the asynchronous QCLS performs worse than the synchronous MRT. In fact, the asynchronous QCLS scheme only seems to guarantee positive throughputs for distances above 40 km. The asynchronous multi-staged QCLS also displays high sensitivity to the UE’s position, performing significantly worse than the asynchronous QCQP. These two plots show that the joint throughput maximization and interference minimization defined in the considered multi-optimization metric is overly conservative in adverse channel conditions. In contrast, synchronous multi-staged QCLS is the least affected by this phenomenon, demonstrating once again the reliable performance obtained through the combination of interference-aware precoding and knowledge of radar scan parameters. In Figure 6c, it can be seen that under particularly good channel conditions, the eNB can serve its users even with simple precoding schemes like synchronous MRT.

Although the considered radar has a significantly higher peak transmit power than LTE systems, the previous Figures suggest that the latter can display reasonable resilience to such strong interference. There are two main reasons behind this phenomenon. The first is that the considered radar system’s signal is very sparse over time, due to its high antenna directivity and rotation, and its very low transmit duty cycle. Secondly, the radar receiver’s interference limit of -6 dB, is much lower than the sensitivity of LTE systems, which can still maintain active links at interference levels above the noise floor given enough eNB-UE proximity.

One important aspect not analysed in the previous plots is the connection latency incurred with each scheme. To illustrate this, for each of the 3-second simulation runs, we calculated the maximum number of consecutive TTIs during which the UE would starve, not receiving any RB allocations due to the risk of the eNB causing harmful interference. We refer to this value as the maximum latency. In Figure 6d we plot the maximum latency CDF across runs, for a radar-eNB distance of 5 km. As can be seen, the CDF for the asynchronous QCLS, multi-staged QCLS, and MRT schemes display non-continuous transitions at the latency of 3 s. The height of this transition corresponds to the proportion of runs where the eNB did not find a feasible precoder over the whole simulation time. This plot provides, therefore, an alternative way to assess the sensitivity of beamforming mechanisms to the UEs’ placements. On the other hand, the asynchronous QCLS and multi-staged QCLS schemes showed superior performance to the synchronous MRT for 85% and 90% of the cases, respectively, as they do not rely on transmission interruptions to avoid interference with the radar antenna lobes. We expect an increase in maximum latency for the synchronous MRT as the radar antenna rotation period increases. This can be explained by the fact that for slow scan patterns, the radar antenna dwell time is higher and therefore the eNB will have to interrupt its transmission for longer time intervals. At a 5 km distance, all schemes except the synchronous multi-staged QCLS show latencies above 150 ms, making them not reliable for latency-sensitive services, such as voice calls [30]. Thus, small cells should only use these techniques for data traffic offloading, maintaining the services with strict Quality-of-
Service (QoS) requirements in an “anchor channel” in licensed spectrum.

D. Throughput for five UEs per small cell

In this test, we measure the total throughput of a small cell with 5 UEs, and the respective radar exclusion zone reductions, in a no estimation noise scenario. The eNB tries to serve all UEs in a balanced manner through the PF scheduling algorithm. However, in cases where the $i_{\text{max}}$ constraint cannot be met, UEs may not get allocated any RBs.

In Figure 7, we can see that the synchronous and asynchronous MRT schemes’ throughput increases comparatively to the single UE case. The reason behind this is that, as the number of UEs increases, the eNB has a higher chance of finding at least one UE it can serve without exceeding the interference limit. Hence, the rate of eNB’s transmission interruptions drops.

E. Robustness to additive estimation noise

In subsection VI-A we demonstrated that the generalization of the proposed beamformers to the robust scenario could be made through the introduction of a constant safety margin. This margin can, in turn, be computed based on $i_{\text{max}}$, the eNB and radar’s transmit powers $P_{\text{eNB}}$ and $P_{\text{radar,Tx}}$, and an estimation error norm distribution, assumed known by the eNB.

In this test, we measure how the performance of a small cell with one UE at a distance of 5 km from the radar is affected by uncertainty in the channel. To do so, we added a random error term $h^i \epsilon_{\text{eNB}}[n] \in \mathbb{C}^N$ to all eNB’s estimates, where each antenna element $i$ follows the distribution

Fig. 6: Cell throughput and UE’s latency for synchronous (sync.) and asynchronous (async.) schemes, one UE per cell and no channel estimation error.

Fig. 7: Cell throughput for 5 UEs per cell.
\( \mathbf{h}^c_{\text{eNB}}[\mathbf{n}] \sim C_N(0, \frac{\sigma^2_{\text{eNB}}}{N_t}) \). This corresponds to an error norm \( e_{\text{eNB}} \) with mean \( \sigma^2_{\text{eNB}} \) characterized by a scaled chi-squared distribution with \( N_t \) degrees of freedom. Summing the error norms across RBs, the inverse CDF of \( \sum_{f} e_{\text{eNB}}[f] \) can be represented as

\[
\mathcal{F}_{\text{eNB}}^{-1}(P) = \frac{\sigma^2_{\text{eNB}}}{N_t} \mathcal{F}_{\chi^2}^{-1}(P, N_t N_{\text{RB}}),
\]

where \( \mathcal{F}_{\chi^2}^{-1}(\cdot) \) is the chi-squared inverse CDF. Substituting (27) in (26), we obtain the following maximum estimation error

\[
\sigma^2_{\text{eNB}} \leq \frac{P_{\text{radar,Tx}}}{\mathcal{F}_{\chi^2}^{-1}(1 - \epsilon, N_t N_{\text{RB}})} F_{\text{eNB}},
\]

for which the target probability of non-interference \( (P) \) can be met. Solving (28) using the parameters for our scenario, as specified in Table I and a \( P \) of 0.99, we find that \( \frac{\sigma^2_{\text{eNB}}}{kTB} \leq 62.3 \) dB, which approximately matches the limit observed in Figure 8a, after which the eNB’s throughput becomes zero. This high upper bound on the allowed \( \sigma^2_{\text{eNB}} \) is the result of the high values that the ratio \( P_{\text{radar,Tx}} / F_{\text{eNB}} \) takes in practice.

We illustrate in Figure 8b the radar station’s incurred INR for different eNB’s estimation error levels. We can observe that: (i) the INR limit of -6 dB is respected for all scenarios; (ii) there is a clear decrease in the caused interference as \( \sigma^2_{\text{eNB}} \) increases; and (iii) for \( \frac{\sigma^2_{\text{eNB}}}{kTB} \) close to the transmit condition limit (26) of 62.3 dB, the depicted curves suffer a sudden interruption, meaning that the INR became \( \infty \) dB. The observation (ii) shows that despite showing a high robustness to channel uncertainty, the proposed robust beamformer is still overly conservative. There are two reasons for this phenomenon. The main one is that the robust beamforming safety margin in (25) is designed for worst case scenarios, where the channel error \( \mathbf{h}^c \) completely aligns with the used precoder. The second reason is that, due to the non-optimality of the proposed precoding approaches, for \( \frac{\sigma^2_{\text{eNB}}}{kTB} \) close to the limit 62.3 dB, the eNB cannot find feasible solutions, which will further decrease the INR.

VIII. CONCLUSIONS

In this work, we quantified the performance of an LTE DL small cell sharing radar spectrum, in terms of cell throughput and incumbent exclusion zone reduction. We extend the state of the art on radar spectrum sharing by considering a multiple antenna scenario, where LTE cells beamforming capabilities are used to mitigate their caused interference to radar services.

Our results show clear benefits of employing cognitive beamforming strategies, in terms of radar exclusion zone reduction. For the cases where the eNB has knowledge and can synchronize with the radar antenna rotation phase, which we called synchronous schemes, beamforming can allow eNBs to operate at almost full capacity even at distances below 5 km. When such synchronization is not possible, which is expected for radar systems with non regular scan patterns, our proposed beamforming schemes still provided exclusion zone reductions of more than 50 km, comparatively to spatial sharing.

One aspect not addressed in this work is the modeling of inter-scan channel decoherence in non-static scenarios. We set as a goal the quantitative characterization of these uncertainties in distinct environments, such as NLOS/LOS, indoor/outdoor, and urban/rural, leveraging data obtained through RF measurement campaigns. We believe this analysis will also determine the suitability of the proposed schemes for mobile LTE terminals.

Another interesting path would be to extend the proposed techniques to the coexistence between LTE and MIMO radars. Such scenario poses new challenges since, unlike mechanically steering radars, MIMO radars may not display repeating scan patterns. Thus, the consideration that all radar antenna configurations can be observed by the LTE system no longer holds, and the interference constraint may not be respected. This issue could possibly be circumvented through increased coordination between radar and communication systems, as is assumed in co-design methodologies [9], [31], or a hybrid scheme that combines instantaneous beamformers like the ones in [8] with the ones suggested in this work.

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APPENDIX A

In order to solve optimization problem (12), we initially transform it to the real valued problem

\[
\min_{\mathbf{x}} \quad \| \mathbf{A} \mathbf{x} - \mathbf{b} \|^2 \\
\text{s.t.} \quad \| \mathbf{x} \|^2 = P_{\epsilon,NB}
\]

where \( \mathbf{x} = [\Re{\{\mathbf{w}\}}^\top, \Im{\{\mathbf{w}\}}^\top]^\top \in \mathbb{R}^{2N_t \times 1}, \mathbf{b} = [0_{1 \times M}, 0_{1 \times (M+1)}]^\top \in \mathbb{R}^{(2M+1) \times 1} \), and

\[
\mathbf{A} = \begin{bmatrix}
\sqrt{\mu_1} \Re{\{\hat{\mathbf{h}}^{\text{EC}}[1]\}} & -\sqrt{\mu_1} \Im{\{\hat{\mathbf{h}}^{\text{EC}}[1]\}} \\
\cdots & \cdots \\
\sqrt{\mu_M} \Re{\{\hat{\mathbf{h}}^{\text{EC}}[M]\}} & -\sqrt{\mu_M} \Im{\{\hat{\mathbf{h}}^{\text{EC}}[M]\}} \\
\sqrt{\mu_{M+1}} \Re{\{\hat{\mathbf{h}}^{\text{UE}}[1]\}} & -\sqrt{\mu_{M+1}} \Im{\{\hat{\mathbf{h}}^{\text{UE}}[1]\}} \\
\cdots & \cdots \\
\sqrt{\mu_M} \Re{\{\hat{\mathbf{h}}^{\text{EC}}[M]\}} & -\sqrt{\mu_M} \Im{\{\hat{\mathbf{h}}^{\text{EC}}[M]\}} \\
\sqrt{\mu_{M+1}} \Re{\{\hat{\mathbf{h}}^{\text{UE}}[1]\}} & -\sqrt{\mu_{M+1}} \Im{\{\hat{\mathbf{h}}^{\text{UE}}[1]\}}
\end{bmatrix}
\]

which can be solved via the eigenvalue decomposition [24],

\[
\begin{bmatrix}
\mathbf{H} \\
\frac{1}{P_{\epsilon,NB}} \frac{g^\top}{g} - \mathbf{I}_{N_t \times N_t}
\end{bmatrix} \mathbf{y} = \lambda \mathbf{y}
\]

where \( \mathbf{H} = \mathbf{A}^\top \mathbf{A} \in \mathbb{R}^{N_t \times N_t} \) and \( \mathbf{g} = \mathbf{A}^\top \mathbf{b} \in \mathbb{R}^{N_t \times 1} \). Let’s denote as \( \lambda_{\text{min}} \) the minimum real solution obtained from (31). Three possible outcomes can happen:

- \( \lambda_{\text{min}} < \min\{\text{eig} (\mathbf{H})\} \), where \( \text{eig}(\mathbf{H}) \) is the array of eigenvalues of \( \mathbf{H} \). The solution to the problem is \( \mathbf{x} = (\mathbf{H} - \lambda \mathbf{I})^{-1} \mathbf{g} \) and is unique.
The (31), $\text{eig}(\mathbf{H})$ inverse and pseudoinverse computations have an algorithmic complexity of $O(N_3)$, which is reasonably low, considering that $N_3$ usually takes a very small value in non-massive MIMO systems.

REFERENCES

[16] 3GPP, “TS 36.211 V8.9.0 Physical channels and modulation (Release 8),” 2009-12.
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